Kieran, C. (2008). Les Débats. *Recherches en didactique des mathématiques, 29*(1), 80-85.

Pour *les Débats* de RDM

I would like to thank Colette Laborde and Rudolf Sträßer for the invitation to follow up on the earlier commentary of Mariolina Bartolini Bussi on "la didactique française" in *les Débats*. There are two issues in particular that were raised by Mariolina to which I would like to react: first, the point of view that the ideas of French didactique have not had a strong impact on the international community and, second, the suggestion that French didactique may be a closed system – not open to the development of ideas from outside the French community.

Mariolina began her commentary by regretting the fact that, "such an important system of theories, methods and attitudes towards didactical research [i.e., French didactique] has such a limited impact on the international audience." I do not believe this to be true. Certainly, in francophone countries, it is not the case. One has only to read the research articles written by, for example, French-speaking Québécois, francophone Belgians, and *didacticiens/ciennes de mathématiques* from the northern half of Africa, to realize that their theoretical frames are heavily influenced by French didactique. But it is also not the case that the impact of French didactique has been limited among Anglo-Saxon and other non-francophone mathematics education researchers.

As an anglophone Montrealer working in a French-Québécois milieu, that of the Université du Québec à Montréal, I have been in close contact during my past 30 years of research with the literature of both linguistic groups. However, the issue is not merely one of language. There is not a strong theoretical tradition in either Canada or the USA. Nor is there a unified school of thought in mathematics education research in these two countries. Thus, while one can speak of French and Italian and German schools of thought in mathematics education research, there is no such single view in the Canadian (or even Québécois) community. The Canadian community of mathematics education researchers is basically quite eclectic with respect to theories and theorizing (cf., Kieran, 2003). That being said, French didactique has clearly had an impact on the mathematics education research of Canadians and other North Americans (see, e.g., the research of G. Barallobres, G. Hanna, P. Herbst, F. Hitt – to name just a few).

For instance, my own research program, and that of my team, has been strongly shaped by the theorizing that has emerged from France. In a project conducted in 1999-2002 with J. Guzmán and H. Squalli on the learning of elementary number-theoretic concepts in an environment that included hand-held calculators, Brousseau's (1997) Théorie des situations didactiques (TSD) served as the frame for the creation and implementation of the didactical sequence. However, I should add that, during the process of data analysis of that study, we encountered the phenomenon of students' referring to the *techniques* they were using with their calculators to obtain certain number combinations – a phenomenon that we could not appropriately analyze with the TSD tools we were using.

They needed to be complemented by other analytical tools. Fortunately, at about the same time, we came across the recent work being done by the French researchers, Artigue, Lagrange, and others, on instrumental approaches to mathematical learning. The theoretical frame developed by Artigue and her research group, which they had adapted from Chevallard's anthropological approach, was well suited to the analysis of our data.

The next project of the research program in 2003-2007, which included my colleagues A. Boileau, F. Hitt, D. Tanguay, as well as J. Guzmán and L. Saldanha, and the project consultants M. Artigue and P. Drijvers, investigated the intertwining development of technique and theory in technology-based algebra learning. Once again, French theoretical frameworks were central to our research. The triad Task-Technique-Theory (Artigue, 2002; Lagrange, 2002) served as our primary research tool not only for constructing the tasks of the study and for gathering the data during the teaching experiments, but also for analyzing that data. Our findings (see, e.g., Kieran & Drijvers, 2006) clearly illustrated the importance of the co-emergence and intertwining of theory and technique in a task setting. Furthermore, although the role of the teacher was not an *a priori* focus of our study, our analyses of student learning suggested that the teaching component was of major importance. In all the classes where our study was carried out, it was the teacher who was pivotal in encouraging the students to struggle with the task, who asked them key questions at appropriate times, who helped them to see the overarching themes within the tasks, who made the instrumental genesis converge to a common set of techniques and insights, and who led the classroom discussions that provoked this convergence through discourse. However, not all of the teachers in the study were equally successful in orchestrating the co-emergence of technique and theory within their students. But, when we later tried to analyze some of the critical factors of teaching practice that could account for differences from one class to the next, we found that the task-technique-theory framework did not provide us with the tools needed to identify these differences – primarily because this theoretical framework is directed toward the learner and not toward the teacher.

Thus, for our most recent project, which is aimed at the analysis of teaching practice in CAS environments, we turned to Trouche's (2004) frame of *instrumental orchestration*, a frame that we felt would be compatible with our task-technique-theory perspective. However, further study of Trouche's frame led us to conclude that our interpretation of instrumental orchestration was broader than his. Let me elaborate briefly.

Trouche (2003) has emphasized that instrumental orchestration is that part of the didactic scenario that is devoted to « *la conduite des instruments* » (p. 43). And, *instruments* are considered to consist of "an artifact component and a psychological component … the psychological component defined through the notion of a *scheme*, [which includes] operative invariants – the implicit knowledge contained in the schemes [such as] theorems-in-action, that is, propositions believed to be true" (Trouche, 2004, pp. 285-6). Thus, within schemes, and therefore also within instruments, conceptual and technical elements are deemed to be intertwined (Drijvers & Trouche, 2007).

However, dominant attention is given to techniques in Trouche's (2004) examples of instrumental orchestration. Because techniques are considered to be the link between tasks and conceptual reflection in instrumental approaches to learning (Lagrange, 2003), one might expect that conceptual reflection/theorizing would be given attention comparable to that of techniques in descriptions of instrumental orchestration. A similar degree of silence is noticed regarding the role played by tasks in the design of instrumental orchestrations. According to Trouche (2004):

> An instrumental orchestration is defined by *didactic configurations* (i.e., the layout of the artifacts available in the environment, with one layout for each stage of the mathematical treatment) and by *exploitation modes* of these configurations. … Instrumental orchestrations can act at several levels: the first level (that of the artifact itself); the second level (a psychological one) of an *instrument* or a *set of instruments*; the third level (a "meta" one) of the *relationship* of a subject with an instrument or a set of instruments. (pp. 296-7)

The second level, which is the psychological one, peaks our interest. Trouche provides an example of this level of instrumental orchestration in his 2004 article. The didactic configuration, which is aimed at promoting socialization of students' actions and productions, involves the use of a view-screen, which in the particular exploitation mode described in the article is piloted by a sherpa-student. According to Trouche, this orchestration favors "collective management of the instrumentation and instrumentalization processes: what a student does with her/his calculator – the traces of her/his activity – is seen by all, allowing the comparison of different instrumented techniques" (p. 298, emphasis added). This comparison of instrumented techniques is considered by Trouche to be a main objective of an instrumental orchestration, "originating from the necessity of orchestration itself." Even if he also refers to secondary objectives of instrumental orchestration, such as "favoring debates within the class" and "developing new relationships between the students and the teacher – about a result, a conjecture, a gesture or a technique" (p. 300), these so-called secondary objectives, that is, those that could involve conceptual issues, are given only passing mention.

Examples of didactic configurations involving technological artifacts such as the view-screen, and exploitation modes involving a sherpa-student under the supervision of a teacher, suggest that, in Trouche's view, instrumental orchestrations are limited to those instrument-oriented activities (in particular, those related to techniques) that are but a part – albeit an important part – of the larger, whole-lesson, « systéme d'exploitation didactique » (Chevallard, 1992). However, our observations of high school teachers in CAS classrooms suggest that instrumental orchestrations are more fluid and encompass a broader range of activity, with much to-ing and fro-ing between using computational and non-computational artifacts, and with a great deal more attention to the conceptual aspects of techniques than are offered in Trouche's (2004) examples. While Trouche notes that a third stage of instrumental orchestration could involve a "colloquium" configuration to discuss conjectures, our experience suggests that student conjectures of both a technical and theoretical nature arise both early and continuously in the ongoing process of instrumental genesis, and need to be taken into consideration in almost all instrumental orchestrations – especially when the tasks are so designed as to encourage

the development of conjectures. This suggests that "colloquium" configurations could usefully be integrated into that which Trouche describes as first and second stages of instrumental orchestration.

In our view, the constructs of didactical configurations and modes of exploitation provide a foundation for thinking about instrumental orchestration, but need to include other components, such as, for example, the socialization of students' emergent theoretical notions (along with their instrumented techniques). In their reaction to Trouche's (2004) article, Hoyles, Noss, and Kent (2004) argued that that which Trouche designates as instrumental orchestration remains schematic, and suggested that the notion of situated abstraction could be used to broaden Trouche's discussion of orchestration. Although our research group is moving in a theoretical direction that is different from that proposed by Hoyles et al., our aim is to elaborate further the frame of instrumental orchestration so as to have not only a theoretical tool that will assist us in coming to a deeper understanding of that which constitutes effective teaching practice with respect to the co-emergence of technique and theory in CAS learning environments, but also a practical research tool for observing and analyzing teaching practice in CAS-supported algebra classrooms.

This brings me back to Mariolina's commentary and the second issue to which I wanted to react. In the same way that our research team is working at introducing something new into the construct of instrumental orchestration, Mariolina has described how M. Maschietto and R. Falcade, Italian researchers who spent a few years in France within a joint doctoral program, both injected supplementary analytical tools into the standard framework of TSD. This issue of outsiders injecting complementary ideas into the theories coming from French didactique led Mariolina to wonder: "What is the place for the development of such ideas in the French community?" She continued with the question: "Do we find examples of theoretical constructs from other research traditions that have had impact on the research developed by French speaking researchers … [or] is the French paradigm working as an obstacle against new ideas, which have not been generated 'within' the paradigm itself?" While not answering these questions directly, she did point out that, in Italian research on classroom processes, they use a "pluralist set of reference theories and methods." Yet, is this not what Lagrange (2005) has tried to do in his integration of the four linked issues of "didactical and epistemological analysis, changes in curricula and practices, tool and mathematics relationship, and design" (p. 148)? Lagrange has argued for including approaches from outside French didactique:

> This calls for a 'multidimensional' approach, consistent with Lagrange et al.'s (2003, p. 239) claim that many research studies or reports of innovation about technology in mathematics education fail to be relevant when they consider only one framework. Theoretical frameworks that will be used … are chosen to provide specific insight into each issue. The first one will be an epistemological and didactical approach of functions in algebra teaching/learning drawn from Kieran (2001). The second and third will be the 'praxeological' and 'instrumental' approaches. As a fourth approach, Yerushalmy's (2001, pp. 183-185) conclusion will provide a framework to consider research design. (Lagrange, 2005, p. 148)

In her concluding remarks, Mariolina proposed that French researchers must decide,

"whether they wish to dialogue with researchers from other research traditions … and if the answer is Yes, they have to distil the substance of French didactics of mathematics." As an aside, I would agree with the inference being suggested here regarding the opaqueness of French theoretical writing. Our francophone graduate students, even some fellow researchers, complain about the difficulty and frustration experienced in trying to understand the dense texts produced by researchers from the French school of didactique. Mariolina then emphasized that French researchers need to "compare their theoretical frameworks, methods, and results [with those of others]." Well, it would seem that steps in this direction are indeed being taken. In 2005, the ReMath project group was formed, consisting of teams of mathematics education researchers from Paris, Grenoble, London, Genoa, Siena, and Athens (Artigue, 2006; Artigue et al., 2006). The ReMath project, which is directed by Artigue, investigated theoretical connectivity in the domain of research on technology in mathematics education; it looked for integrating perspectives in terms of theoretical frameworks. The resulting Integrative Theoretical Frame that was generated by the ReMath group can be seen as a complementary conceptual tool that bridges the gap between different research contexts. It provides "a common structure for describing and comparing different approaches and perspectives in the field of mathematics education" (Artigue et al., 2006, p. 55). Even if this project concerns primarily research in technological learning environments, it provides a significant indication of an emerging openness among French researchers to theoretical frameworks from the outside.

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