Régularités via Maple

$$M_n = \frac{a_n}{b_n}$$

$$a_n = b_{n+1}$$

$$6 - \frac{a_n}{b_n} = \frac{5^n}{b_n}$$

$$6b_n - a_n = 5^n$$

$$b_{n+1} = a_n = 6b_n - 5^n$$

Forme des dénominateurs

$$b_1 = 6b_0 - 5^0$$

$$b_2 = 6b_1 - 5^1 = 6\left(6b_0 - 5^0\right) - 5^1 = 6^2 b_0 - \left(6^1 5^0 + 6^0 5^1\right)$$

$$b_3 = 6b_2 - 5^2 = 6\left[6^2 b_0 - \left(6^1 5^0 + 6^0 5^1\right)\right] - 5^2$$

$$= 6^3 b_0 - \left(6^2 5^0 + 6^1 5^1 + 6^0 5^2\right)$$

$$b_4 = 6b_3 - 5^3 = 6\left[6^3 b_0 - \left(6^2 5^0 + 6^1 5^1 + 6^0 5^2\right)\right] - 5^3$$

$$= 6^4 b_0 - \left(6^3 5^0 + 6^2 5^1 + 6^1 5^2 + 6^0 5^3\right)$$

$$\cdots$$

$$b_n = 6^n b_0 - \left(6^{n-1} 5^0 + 6^{n-2} 5^2 + \cdots + 6^1 5^{n-2} + 6^0 5^{n-1}\right)$$

Forme de la suite de Muller

$$b_n = 6^n b_0 - \left(6^{n-1}5^0 + 6^{n-2}5^1 + \cdots + 6^1 5^{n-2} + 6^0 5^{n-1}\right)$$

$$= 6^n b_0 - 6^{n-1}\left[\left(\frac{5}{6}\right)^0 + \left(\frac{5}{6}\right)^1 + \cdots + \left(\frac{5}{6}\right)^{n-2} + \left(\frac{5}{6}\right)^{n-1}\right]$$

$$= 6^n b_0 - 6^{n-1}\left[\frac{1-\left(\frac{5}{6}\right)^n}{1-\left(\frac{5}{6}\right)}\right] = 6^n b_0 - 6^n\left[1-\left(\frac{5}{6}\right)^n\right]$$

$$= 6^n b_0 - 6^n + 5^n = 5^n + 6^n \quad car \;\; b_0 = 2$$

$$M_n = \frac{a_n}{b_n} = \frac{b_{n+1}}{b_n} = \frac{5^{n+1} + 6^{n+1}}{5^n + 6^n}$$

Limite de la suite de Muller

$$M_n = \frac{5^{n+1} + 6^{n+1}}{5^n + 6^n} = \frac{5\left(\frac{5}{6}\right)^n + 6\left(\frac{6}{6}\right)^n}{\left(\frac{5}{6}\right)^n + \left(\frac{6}{6}\right)^n} \rightarrow \frac{5 \cdot 0 + 6 \cdot 1}{0+1} = 6$$