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AN "INSTRUMENTAL APPROACH" TO STUDY THE INTEGRATION OF A COMPUTER TOOL INTO MATHEMATICS TEACHING: THE CASE OF SPREADSHEETS

ABSTRACT. This article reports on research focused on the integration of a specific computer tool, the spreadsheet, into mathematics teaching. After presenting some important results obtained by research in this area, we revisit these in the light of an instrumental approach, which we perceive as essential to analyse the construction of mathematical meanings in spreadsheet environments and to understand better the questions of technological integration. Then, these theoretical elements are used in order to design an exploratory experiment with grade 7 pupils and analyse its results.

KEY WORDS: instrumental approach, spreadsheet and mathematics education, transition arithmetic-algebra

1. INTRODUCTION

Nowadays, there is a firm encouragement to integrate the technologies of information and communication into the curricula and educational practices in France. In mathematics, teachers are expected to use, beyond calculators, Internet and various software: dynamic geometry and computer algebra software, spreadsheets, etc. Our research deals with spreadsheets and was motivated by the following observations:

- Although spreadsheets have been part of the French syllabus since 1997 – they first appeared in the middle school syllabus then in the high school one – their use in mathematics still remains very marginal.
- The quantity of pedagogical spreadsheet resources, in textbooks, publications or web sites has significantly increased without evident impact on the integration of this technology.

These phenomena raise many questions from both a theoretical and a practical point of view:

- What do we know about the potentialities of spreadsheets for mathematics education and about the necessary conditions for their achievement?
- Does their integration generate specific problems and, if it does, what are they?
- What are the characteristics of existing resources? Up to what point can they help the integration intended by the institution?
- How do teachers, who really have integrated spreadsheets, work? How did their practices develop and how do they keep on evolving?

An inventory of didactic research in this area shows the importance given by researchers to the potential of the spreadsheet for the teaching and learning of *algebra*. Yet, as one can also notice, these studies seem to pay little attention to instrumental issues whose importance for technological integration has been recently evidenced in research about CAS,[1] a technology that, as spreadsheets, was not initially designed for educational purposes. This is the reason why we decided to analyse what this theoretical framework could afford in the case of spreadsheet technology. In Section 2 of this paper, we present this study and its main results; we then describe and analyse an exploratory design inspired by its ideas.

2. BETWEEN ARITHMETIC AND ALGEBRA: A THEORETICAL SPACE FOR SPREADSHEETS

In paper–pencil environment, the difficulties of algebraic learning have already been extensively investigated and often related to different analyses: procedural/ structural duality (Sfard, 1991), semantic/syntactic difficulty (Drouhard, 1992), arithmetic/algebra relationships (Vergnaud et al., 1988; Vergnaud, 1989/90). Thus, the position of algebra with respect to arithmetic has often been seen in terms of *ruptures*: discontinuities/false continuities, (Bednarz and Janvier, 2001) or in terms of four important transitions (Rojano, 2001):

1. from what is numeric (digital) or verbal towards what is symbolic;
2. from what is specific towards what is general;
3. from work with what is known towards work with unknowns;

4. from intuitive processes (not algebraic) towards school rigorous (algebraic) processes.

The proceedings of the 12th ICMI Study (Chick et al., 2001) summarises research advances in this area. Regarding these researches, studies in spreadsheet environments take different approaches, as we will see now.

2.1. *Different Visions... and Common Points*

The use of spreadsheets in mathematics teaching has been studied for instance by Ainley (1999), Ainley et al. (1999, 2003), Arzarello et al. (1994, 2001), Capponi (1999, 2000), Dettori et al. (1995), Rojano and Sutherland (1997), and Rojano (1996, 2001). All these studies relate to the learning of algebra at elementary stages and generally give spreadsheets a positive role in this learning. They all emphasise the ambiguous or the hybrid (arithmetico-algebraic) status of spreadsheets but do not draw from it the same outcomes.

Some researchers use spreadsheets to 'face' these difficulties or cognitive obstacles. Most of them situate their work in a *constructivist* framework and show how *spreadsheets functionalities* (Capponi, Dettori et al.) can help to overcome some of the difficulties listed above (Rojano and Sutherland). For instance, Capponi (1999, 2000) develops a detailed analysis of the tool characteristics considering three dimensions:

- Their functional and structural features such as the screen display, the warrants for edition and the systems of designation.
- The knowledge they involve such as the solving processes, the status of the equal sign, the status of letters and produced objects.
- The pupils' errors when using spreadsheets in writing, interpreting or copying formulas.

For each element of these dimensions, he evidences the spreadsheet intermediate character. For instance, the screen displays two levels: the sheet of calculation, which is temporarily apparent and the tables of numerical values resulting from these calculations, which are most of the time visible. Capponi points out that tables favour arithmetic work, whereas the sheet of calculations highlights the underlying formulas and algebraic concepts. In their work, Dettori et al. also study the question of the knowledge involved in spreadsheets but insist more on the limitations this knowledge induces. For instance, the equal sign in spreadsheets is, for Dettori et al. (1995, p. 265) "actually the assignment of a computed value to a cell, while the equal sign in algebra represents a relation". For Capponi, this is not exactly true as the algebraic status of a relation of equivalence is also present for the equal sign in spreadsheets through mathematical functions such as the logical one "IF(a = b; ;)". More recently, some researchers have developed a similar approach within a *socio-cultural* frame. This is the case with Arzarello and his colleagues, who consider spreadsheets as systems of social interactions, where teachers and pupils build a new socially shared language: the algebraic one.

Some other researchers use spreadsheets to go beyond the vision of pupils in a transition "from one stage (of knowledge) to another [and] propose instead the need for multiple narratives in order to capture the complexity of the learning process" (Ainley, 1999). They adopt a different perspective by showing the existence of "*emergent algebraic*" knowledge in Year six children familiar with spreadsheets (Ainley, 1999) and stress the importance of situation design for developing meaningful algebra (Ainley et al., 2003). For Ainley (1999), spreadsheets ambiguities provide interesting opportunities to introduce to, and to show the need for, a symbolic notation. For instance, the cell references of spreadsheets appear ambiguous in a powerful way: "When a cell reference is used within a formula, the cell in question may contain a number, or another formula" (Ainley, 1999). In the same way, our work extends the analyses of the cell references and formulas (see Section 3.2.1).

In conclusion, we can say that spreadsheets appear as good tools for semiotic mediation, or as hybrid tools living in a transitional world between arithmetic and algebra. A didactical issue is that this position seems 'ideal' (under certain conditions) to help the transition from arithmetic to algebra (Rojano and Sutherland, 1997) or to produce interesting ambiguities for learning about and using algebraic ideas (Ainley, 1999). How? On which potentialities do researchers focus on? We discuss this in the next paragraph.

2.2. *The Potentialities of Spreadsheets for the Learning of Algebra*

Spreadsheets' potentialities rely on their constraints: constraints of communication, of symbolism, of organisation; and on the new action modalities they put forward: interplay of various languages, multiple representations and interactivity. Some researchers focus on spreadsheets' symbolic potential (through the notions of variable or formula); others focus on their strategic and methodological potential (through the organisation of the sheet and the planning of the work, which both require the user to anticipate a spreadsheet's behaviour).

2.2.1. *The Symbolic Lever*

Communicating with a spreadsheet requires that pupils use an interactive algebra-like language, which focuses their attention on a rigorous syntax. This is why it is said that spreadsheets help to translate a problem by means of an algebraic code (see the example in Section 2.2.2). Furthermore, this constraint does not come from a didactical contract, as it is usually the case in paper pencil environment, but from the structure of the tool itself. For Ainley, when beginning algebra, "it is very difficult to have any sense of the purpose of algebra, of what it is that algebra is useful for" (Ainley, 1999, p. 10). Spreadsheets' constraint of communication allows children to "appreciate the need for an algebra-like notation" and provides "new ways for children to be introduced to" it (Ainley, 1999, p. 9).

In a wider way, several semiotic registers can be present on the screen due to the spreadsheet capacities of representations:

- The register of the natural language: one can edit some text, remaining close to the context of the problem, keeping in mind what she/he calculates.
- The register of formulas: one can express the relations between cells.
- The numerical register: with the cells representing data of the problem or results from calculations.
- The graphical register: spreadsheets include a graphic application that allows the user to draw several types of graphical representations dynamically linked to the numerical data.
- And a register specific to spreadsheets, which relates to the numerical register but at the same time, calls up the notion of variable. This 'numerical-variable' register offers the possibility of varying specific numerical values to get different results (for example, to solve a problem by 'trial and error').

This multiplicity of available representations can support the connection between different registers, which is seen more and more as playing an essential role in conceptualisation. In the case of algebra, it can help pupils to find the relations between all the data. It also provides more means of control. For example, the numerical feedback obtained while working on a formula allows pupils to experiment, conjecture and may help them to find their errors. Hence, the work is richer than in paper–pencil environment: spreadsheets enhancing the sense of algebraic symbolism, playing on one of the theoretical difficulties mentioned above, the complex semantic/syntactic relationships in algebra.

2.2.2 *The Methodological Lever*

A fair number of problems (optimisation, equations) can be solved with spreadsheets by using a method which is close to the classical paper–pencil 'trial and refinement' (T/R) method. For some researchers, using this T/R method, instead of writing algebraic relations as equations, shows spreadsheets incapacity to complete the learning of algebra: "Our analysis emphasises that the spreadsheet can be useful to introduce some elements of algebra, but that its results are inadequate, if not misleading, for a deep learning of the fundamental aspects of algebra" (Dettori et al., 1995, p. 262). On the contrary, Ainley (1999) stresses spreadsheets' positive role when used to solve problems through trial and improvement processes. For Rojano and Sutherland too, T/R methods are positive because they allow pupils to progress from their arithmetical intuitive methods towards more algebraic ones: "pupils' informal processes can be used as a basis to build up 'more algebraic' methods of solving problems when working in a spreadsheet environment" (Rojano and Sutherland, 1997, p. 72). Let us take the example they give, the "Chocolates Problem" (p. 75):

> 100 chocolates are distributed amongst 3 groups of children. The second group receives 4 times the number of chocolates as the first group. The third group receives 10 chocolates more than the second group. How many chocolates does each group receive?

With this problem, Rojano and Sutherland (1997, p. 78) show that pupils "bring into play informal T/R stategies, which have one aspect in common with the algebraic (Cartesian) method". In our work, we analysed this method and compared its use in paper–pencil and in spreadsheet environments. We showed that spreadsheets bring some specificity making the method even closer to an algebraic one. We describe below these three methods.

An arithmetic resolution can be the T/R method (the resolution by analysis/synthesis turns out impossible here, see Rojano and Sutherland, 1997, p. 77). For instance, pupils can try with 33 (100 approximately divided in 3 parts) for the first group, see that it is too much – the second would have 132 – then try another value – less than 33 – and so on, until they find that 10 gives a correct solution: the first group has 10 chocolates, the second 40 and the third 50.

An algebraic resolution leads, for instance, to the system:

$$\begin{cases} y = 4x, \\ z = y + 10, \\ x + y + z = 100 \end{cases}$$

(where x, y , z are the respective numbers of chocolates of the three groups) and to the equation: $x + 4x + (4x + 10) = 100$, with the solution: $x = 10$, $y = 40$ and $z = 50$.

What would be a spreadsheet resolution?

Figure 1 is an example of a sheet that pupils can create and use. In this resolution, by transferring the problem's data into different cells and translating the relations between these data by relations between cells (formulas), some intermediate expressions emerge, which are very close to the equations algebra would have led to (Rojano and Sutherland, 1997): in the cell B2, the formula representing group 2 " = 4*A2" is very close to $y = 4x$. The same for group 3, the formula "= B2 + 10" is very close to $z = y + 10$. Yet, at the same time, the global resolution process is very close to the T/R one described above except that calculations and trials are here structured and automated. Indeed, in this example, we see that the resolution demands to structure the problem: it is necessary to organise a sheet of calculations, thus to identify the data, to transfer them in cells, to identify the relations between the data, to translate them by formulas. In consequence, pupils can better take into account all the data, operate with the unknown, and identify the intermediary relations between the data (converted in the sheet into formulas linking various cells).

	A	B	C	D
1	group 1	group 2	group 3	total
2		=4*A2	=B2+10	=A2+B2+C2

Figure 1.

In short, we can say: *spreadsheets add an algebraic organisation to an arithmetic resolution.*

2.3. Conclusion: Potentialities but also Limitations...

Considering such strong potentialities and benefits offered by spreadsheets, we wonder why is their use still such limited in mathematics education. Actually, for some researchers, the intermediary position of spreadsheets can act as a restriction for learning algebra. Some authors, such as Capponi or Dettori et al. stress the point that spreadsheets' positive role is not automatic. In particular, the arithmetical side is seen as a limitation for

> a deep learning of the fundamental aspects of algebra (...) due to several factors: spreadsheets deal only with numbers, or addresses of numbers, and functions; algebraic variables and relations can not be directly handled. (Dettori et al., 1995)

The intermediary position of spreadsheets can even maintain the pupil *completely on the arithmetical side* (Capponi, 1999, 2000). For instance, one can edit a formula using C2 without having seen any symbolism: just by clicking on C2 and, doing so, pointing out a number; the intention and action being to perform a numerical operation and not to write an algebraic formula![2] For Capponi, benefiting from the features of a spreadsheet requires that users already have some algebraic knowledge such as understanding the notions of formula, of variable... Pupils' difficulties in using a spreadsheet show their needs in this area: their work remains at a numerical level (tables of values, numbers and operations) and does not reach the level of the algebraic treatment (dynamic sheet, formulas). We perceive here how central the question of linking tool features with algebraic concepts is. What to learn, what to teach first and in which progression? The teacher shall find his own answers to integrate spreadsheets into the mathematical sequences he already built. Dettori et al. (1995) also point out teachers' role:

> Further steps toward a real learning of basic algebra can be made through a reflection, strongly guided by the teacher, on the resolution model implemented by means of the spreadsheet. In fact, the teacher's role appears essential. (Dettori et al., 1995)

So, we must analyse more precisely the technical characteristics and functionalities of the tool, their possible impact on conceptualisation, and their link with mathematics so that spreadsheets' potentialities become actual. Capponi's work echoes with a theoretical frame that already arose questions of instrumentation and its relations with conceptualisation in CAS environments (Guin and Trouche, 1999; Lagrange, 1999; Artigue, 2002). These issues lead us directly to the question of instrumentation, which allowed us to better understand and settle the problems of technological integration, as we will see now.

3. AN "INSTRUMENTAL APPROACH" IN DIDACTICS

A new approach in didactics recently developed in the context of CAS (Lagrange, 1999; Artigue, 2002; Trouche, 2003a, b; for a synthesis see Guin et al., 2004). As mentioned in the introduction, this theoretical work seems interesting for answering the questions above in the spreadsheet case.

3.1. *The General Framework*

The framework that inspired this part of our work is a socio-cultural framework based on two approaches (Artigue, 2002, p. 245):

- The anthropological approach (Chevallard, 1998; Bosch and Chevallard, 1999) give tools to approach personal and institutional practices and thus to take into account the institutional dimensions of technological integration. These latter practices are described in terms of tasks, techniques and theories.[3] This framework puts the accent on the role played by techniques in the building of mathematical knowledge.
- The theory of instrumentation (Rabardel, 1993, 1999; Vérillon and Rabardel, 1995), which is a psychological and socio-cultural frame, developed in cognitive ergonomics. This theory provides new views concerning learning processes in complex technological environments. Its first key idea is the distinction artefact/instrument. Within the activity of a subject (Vérillon and Rabardel, 1995) a material or psychological *artefact*[4] becomes an *instrument* through a progressive individual *genesis*, the so called *instrumental genesis*. This latter evolves in two interrelated directions. First, towards the tool itself, it

is the *instrumentalisation* process (for example, various potentialities of the artefact are progressively discovered, or possibly transformed in personal ways). Second, towards the subject, it is the *instrumentation* process (for example, using a graphic calculator to represent a function may play on pupils conceptualisations of the notion of limit). Thus, the idea of instrumental genesis reflects the fact that using a tool is not a one-way process, there is dialectic between the subject acting on his/her personal instrument and the instrument acting on the subject's thinking.[5] The process of instrumental genesis is described by Trouche as follows:

> This organ construction, named instrumental genesis, is a complex process, needing time, and linked to the tool characteristics (its potentialities and its constraints) and to the subject's activity, his/her knowledge, and former method of working. (Trouche, 2003a)

An example describing the instrumental genesis in a computer algebra tool environment is given in Guin and Trouche (1999).

Research has shown that instrumental geneses are much more complex than one could expect and, up to now, mainly underestimated by educational systems (Artigue, 2002). In a teaching perspective, this raises the issue of their didactic accompaniment:

- From an instrumental point of view, a given artefact leads to various instruments for various subjects. Moreover, an artefact is actually inscribed in a system of various artefacts (like spreadsheets, which are software used through another artefact: the computer) and can relate to external artefacts: other software, calculators... Thus, for a given artefact, the teacher must consider various geneses enrolled in complex systems of various interacting instruments.
- From an anthropological point of view, teachers' tasks are complicated by the use of a new environment since mathematical knowledge and conceptualisation are tightly dependent on techniques (Lagrange, 2000). Indeed, like in paper–pencil environment, the teacher has to organise the tasks so that the pupil elaborates some techniques of resolution as bases on which he can develop his comprehension of a concept (Lagrange, 2000). But new techniques emerge through the use of new tools and interact with the usual ones. So using a new

environment implies a reconsideration of teachers' mathematical organisations (sets of tasks, techniques and theories) usually managed in paper–pencil environment.

Recently, Trouche (2003a, b) introduced the term of instrumental orchestration to approach the (necessary) external guidance by the teacher of the students' instrumental geneses:

> An instrumental orchestration is defined by *didactic configurations* (i.e. layout of the tools available in the environment, one layout for each stage of the mathematical treatment) and by *exploitation modes* of these configurations. (Trouche, 2003a)

This aims at providing tools for analysing teachers' setting up of instrumented activities, and the way they take into account the various instrumental geneses at stake; in other words the environmental organisation "i.e. the organisation of the students' and/ or teachers' work space and time" (Trouche, 2003a, p. 13). One can find some detailed examples of instrumental orchestration in Guin et al. (2004).

In conclusion, this instrumental frame allows analysing instrumental geneses (both personal and institutional ones) as well as studying their didactic assistance. These issues play an essential role in the problems of integrating computer technologies into education, but were up to now rarely considered in school institutions and in teacher training. The instrumental approach in didactics leads to a more critical vision of tools potentialities for teaching and learning, showing a complexity which contrasts with the 'easy' separation technical/ conceptual and with the hypothesis of 'natural' integration initially claimed in literature and research (Artigue, 2002). As stressed by this author, the relationships between the technical and the conceptual part of mathematics should be thought in terms of dialectics rather than opposition. These observations and conclusions apply to the case of spreadsheets. Thus, as regards to traditional teaching, spreadsheets use for learning algebra does not settle any more as: 'the learning of the one is there to remedy the incapacity of the other one' but rather in terms of 'transmission of the elements of a technical/ conceptual dialectics' fitted in a certain mathematical culture, which *is precisely* instrumented (Rabardel, 1999; Lagrange, 2000). This vision also raises questions about what is an algebraic culture in a spreadsheet environment, which is connected somehow to the point of view of Ainley (see Section 2.1). Therefore, we have to analyse more precisely what instrumentation means in that case and how it can develop. As we will see, the theoretical frame described above intervenes in three ways:

- in studying the way a spreadsheet can become a mathematical instrument for pupils through the analysis of spreadsheets' potentialities and constraints of use (see Section 3.2);
- in studying the dialectic relationships between conceptual work in algebra and technical work in spreadsheet (see Section 3.2);
- in exploring these issues whilst taking into account the institutional dimensions of learning processes in school in order to explain teachers' reluctance in integrating spreadsheets into their teaching practices (see Section 4).

3.2. *The Case of Spreadsheets: The Needs of the Instrumental Genesis*

The previous framework leads us to analyse spreadsheets potentialities and constraints of use and to question the relations between spreadsheet objects/methods and paper–pencil ones. What solving processes, methods or techniques does the spreadsheet favour? How do the usual algebraic objects live in this environment, especially those already pointed as problematic in the paper–pencil environment? What are the new objects introduced by this technology? How can instrumental genesis be assisted and guided?

We present here two main results from this ''instrumental'' analysis. The first one deals with the new objects introduced by the tool, the second deals with instrumental genesis and its underestimation in the research literature.

3.2.1. *The Usual and New Algebraic Objects in Spreadsheets*

These new objects result from spreadsheets new possibilities, their constraints and the gestures that their use requires. We describe here some examples of new objects.

Let us start by the study of two cells connected by a formula. We want to strengthen the ambiguity of cell references that Ainley (1999) and Capponi (1999) have already underlined by providing evidence for the existence of a new object which we will call the '*cell variable*'. In a paper–pencil environment, variables in formulas are written by means of symbols (a letter generally for the school levels concerned here). This variable 'letter' is connected to a set of possible values (numerical here) and exists *in reference* to this set. In a spreadsheet, let us take the example of a formula for square numbers. Figure 2 shows a cell argument A2 and a cell where the formula was edited B2, referring to this cell argument.

Here again the variable is written with symbols (those of the spreadsheet language) and exists, as with paper and pencil, in reference to a set of possible values. But this referent set (abstract or materialised by a particular value, e.g. 5 in Figure 2) appears here through an intermediary, the cell argument A2, which is both:

- an abstract, general reference: it represents the variable (indeed, the formula does refer to it, making it play the role of variable);
- a particular concrete reference: it is here a number (in case nothing is edited there, some spreadsheets attribute the value 0);
- a geographic reference (it is a spatial address on the sheet);
- a material reference (it is a compartment of the grid, some pupils can see it as a box).

So, where in paper–pencil, we stick a set of values, a cell argument overlaps here, embarking with it, besides the abstract/general representation, three other representations without any equivalent in paper–pencil. Henceforth, to remind ourselves of these differences, we shall call it '*cell variable*' (Figure 3).

Let us notice that the cell B2 has a double, or two-faced, status: it refers both to a formula and to a possible variable for a new formula in another cell! For more examples, let us add now to the previous situation some of the most interesting spreadsheet features, the 'filling down' (or '*re-copy*'), the '*assigning names*' and the '*automatic re-computing*'.

The spreadsheet function of re-copy complicates the situation: for instance, the formula in B2 can be re-copied automatically by dragging down the handle of re-copy generating a 'variable column' which is another object different from the previous one.

Then, it is also possible to assign a name to a group of cells, for instance *n* for the group A2:A5, and use this name in a formula for

	A	B
1		
2	5	=A2^2

Figure 2. A2 is the cell argument; B2 calculates the square of the value in A2.

instance "$=n\hat{}2$" in B2. By doing so, we generate another notion of variable: this time, the variable is n and the intermediary is a finite number of 'cells arguments', each of them having the characteristics of a variable-cell. Yet, this 'variable-group' is not a mere group of variable-cells placed side by side, the fact that they are linked by the same name n adds a new dimension to this notion of variable: the numeric multiplicity. This dimension carries along a conception of variable very close to the traditional one. But, as we could observe it, afterwards, this functionality has never been used in the pedagogical resources for teachers we analysed up to now!

Finally, the functionality of automatic re-computation and the dynamic aspect of the sheet are particularly interesting with formulas including absolute references (like \$A\$1...). The automatic re-computation of formulas (when a value is changed) distinguishes absolute/relative reference, which relates to the distinction parameters/variables: here, the notion of parameter, as a variable of the problem, emerges not only through a cell but also precisely through the gesture of automatic re-computation of the sheet.

In the same way, the formulas live in a specific way in spreadsheet. By a similar study, we can analyse them through each of the previous functionalities. For instance, if a formula is copied by dragging, its usual operational invariance is not translated by a syntactic invariance in the spreadsheet; in the previous example, the formula of the square, if dragged down the column, becomes A2ˆ2, A3ˆ2, A4ˆ2 etc. (Figure 4)

The symbolic writing of the formula varies from line to line because of relative references. In a paper–pencil environment, one could see this task as inputting diverse values to the variable of the same formula. However, changes here do not concern numerical values: variables are changed into other variables, while referring to the same equivalent paper–pencil variable. This is why we labelled it a 'column-formula'. In the previous situation, it is not simply a change

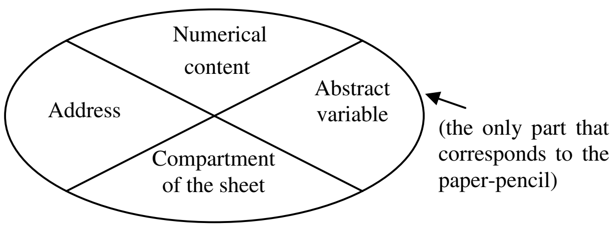


Figure 3. The 'cell variable'.

	A	B	
1	5	=A1^2	
2	17	=A2^2	
3	-11	=A3^2	

Figure 4.

in the writing of the variable from line to line, but a change of the cell-variable it refers to. For instance the formula above in B1 does not refer to the cell A2 nor A3, but only A1; in B2, the cell argument has changed, with all that it embarks: general variable (the same for every line), numerical (digital) contents (different for every line), material box/compartment of the table (different for every line), address (different for every line).

The succession of calculations, found by applying the same symbolic writing to various values in a paper–pencil environment, becomes, in a spreadsheet, a reproduction of formulas written differently, showing various results, but having the same structure, i.e. referring to the same mathematics formula. Yet, as the 'column-formula' is displayed temporary, we could think it will have little impact on pupils who are interested only in the results. Moreover, if the activity concerns work on formulas, as the operative invariance of the formula is not translated by a syntactic invariance, then how does this invariance make sense for pupils? Can it be through the gesture of dragging? What roles do these gestures play in the recognition of this invariant? Are they sufficient? What is the teacher's role for 'grasping' and understanding this? How are these questions taken into account in the research literature? While reviewing research on spreadsheets, our analyses revealed the existence of many implicit elements. This is the point we will illustrate now.

3.2.2. *The Existence of Implicit Elements in the Research Literature*

Each activity has its own mathematical pre-requisites and objectives. Beside these mathematical elements, activities with spreadsheet also carry some technological pre-requisites. These are not generally explicit in texts, yet, they are definitely necessary so that the task can be properly approached and the actual activity be the activity aimed at. Let us give an illustrative example.

In Arzarello et al. (2001), 12/13 year-old pupils were given the task to name a general odd number. In the paper, we do not know the details of the work, the context of the situation, the didactic contract and so on. The only information we have is that the professor gives a first column of successive integers (Figure 5) with, as a first task, to name this column (*n* for example), he then expects the production of $2n+1$ for the name of column B.

The task is thus transformed in:

> From the numbers of column A, find a formula, which generates odd numbers in column B.

The pupil can make a sequence of operations on the numbers of column A: for example do A3 + 1, by clicking A3, to make the operation 2 + 1 and obtain the odd 3, but she/he can also do 2*A3 + 1 to obtain the odd number 5. Each time the spreadsheet will receive a formula corresponding to these operations, perceived (or not) by the pupil who will have then a numeric feedback allowing him/her to check his/her work (is it an odd number or not?).

The task clearly requires that pupils find a *general* name or formula, so excludes a resolution such as in the Figure 6, which also gives odd numbers.

The teacher expects without any doubt, the same formula to be copied downward every line to reach his goal: obtaining the structure $2n+1$. Here, the feature of copying down is thus essential. But how does the pupil understand that it is the same formula, which must be copied down if she/he is just learning the notion of formula in this context, all the more as spreadsheet will not show *the same* formula (see Section 3.2.1)? We can make the hypothesis that it is not the word "general" of the task that is going to help her/him but rather 'the obligation' to use the feature of copying down. In the sheet, Figure 6, this had not been used. It is the resort to this spreadsheet feature that can lead pupils to the expected formula.

How is this constraint created? Is it only by the words "general name" or "general formula"? Is it the didactic contract with the teacher that highlights the dragging down feature? Is it the place of

	A	B
1	0	
2	1	
3	2	
4	3	
5	...	

Figure 5.

the exercise in the teaching sequence? We do not know. Furthermore, even supposing that the pupil uses the dragging downward of single formula, the exercise can still not reach its goal because another possible resolution can be the one shown in Figure 7, where the formula "= A1 + A2" copied downward does supply a solution by adding two consecutive integers.

What is the problem here (for the teacher, not for the mathematics or for spreadsheet)? The problem is that we used several lines in the same formula. So the constraint of using the spreadsheet is even more precise. It is about using the feature of *dragging* downwards *a formula* that is *only created from the corresponding line* of column A, (i.e. use in the B1 formula only the cell A1). This constraint is here again absent in the task of the exercise. So, here again, similar questions arise: who/what leads pupils to act in the foreseen direction? How is this constraint created? What is the teacher's action before/during/after the activity?

In order to delimit and understand the problems of integrating spreadsheets into teachers' practices, we wonder whether the resources available to the teachers take into account these implicit components. If they do, then how do they suggest the teacher manages them? Is it by way of the progression, or by playing on some effects of the didactical contract with pupils? In order to favour the learning of algebra through spreadsheets use, all researchers underline the importance of the situations at stake. But which didactical variables are they playing on? Whereas we can point out the mathematical variables used in their situations, the 'instrumental' ones (that is regarding the tool features) mostly remain implicit. Yet, if these elements are not regarded, they might generate several misunderstandings, pupils using spreadsheets in ways 'other' than the way expected by the teacher. The example given above illustrates this. The organisation of the learning (didactical and mathematical), the way the tool is introduced, its links to mathematics, the techniques taught, their links with the mathematical techniques already learned in a paper–pencil environment (or to be learned), the role of the teacher in

	A	B
1	0	
2	1	=A2
3	2	=A2 + A3
4	3	
5	...	

Figure 6.

	A	B
1	0	=A1+A2
2	1	
3	2	
4	3	
5	...	

Figure 7.

this and its didactic managements; all these elements are missing. For instance, how and when does the teacher introduce into the learning the technical important specificity of spreadsheets, like the functionality of dragging we have just seen? The question of linking the tool features with the mathematical concepts arises here again, revealing that the work will be different from the one in paper–pencil environment. What exactly are these differences and what impact could they have on the learning of algebra?

3.3. *Conclusion*

In spite of an apparent simplicity of use, the tool generates some complexity: new objects are created, usual objects are modified and new action modalities are available. At the very moment when the pupil begins the transition towards algebra, when she/he must both give new status to known objects and change his/her methods of resolution, several elements specific to spreadsheets intermingle and interfere with the concepts of variable, unknown, formula, equation... Do these interferences have a positive, negative or negligible influence on the expected conceptualisations? Regarding teachers' tasks, which questions does the beginning of algebra with spreadsheets raise? How can they be solved? Could their underestimation explain certain failures of integration?

4. TOWARDS THE PRACTICES: AN EXPLORATORY EXPERIMENT

To tackle the questions listed above we carried out an exploratory experiment in which we tried to make pupils approach some algebraic knowledge *using* the spreadsheet.

4.1. *The Experimentation*

4.1.1. *Methodology*

We experimented in the last term of a school year with a class of grade 7 pupils who had not received any teaching of algebra; let us name it the class "A", and its teacher "Dan". I also had that year the possibility to experiment in my own class, let us name it class "B". It was a precious opportunity because class B had quite a different profile, rather atypical (see details in Section 4.2.1). This allowed me to test the robustness of the results obtained from the case study. Besides, my spreadsheets sessions taking place before those of class A, they permitted to 'pre-test' the sessions before carrying them out in the class A. This additional class also enabled us to observe unexpected astonishing similarities (see Section 4.2.1).

- The object of the sessions. Starting from the theoretical study described above, we built up a progression intertwining the development of algebraic and spreadsheet knowledge. We then experimented in the computer lab a sequence of five sessions that Dan and I (as a teacher) elaborated on the basis of this progression. While being familiar with the educational use of dynamic geometry software we were both integrating spreadsheet for the first time into our teaching and thus had to find suitable contexts and tasks corresponding to the planned gradation. Hence, the sequence was not fixed in advance but progressively developed along the sessions considering what Dan and I felt about our pupils' knowledge.
- The observables. In both classes pupils worked in pairs. I observed Dan's sessions and copied all pupils' written papers and spreadsheet files. As a researcher, I also collected the various remarks or personal feelings we exchanged about each session and about the elaboration of the tasks. Finally, I interviewed Dan at the end of the entire sequence.

4.1.2. *The 'Maths and Spreadsheet' Progression in this Teaching Sequence*

The sessions have been built on the theoretical progression composed of three stages given in Table I.

In the next section, we present some interesting results of this exploratory experimentation.

TABLE I

Algebraic knowledge	Spreadsheet functionalities
1. Meeting formulas	
Entering symbolism, introducing letters	
• Discover the existence of relations between numbers	• Play on the dynamic relation between two cells
• Identify these relations as types of formulas	• Spreadsheet objects: ‘cell-variables’, ‘cell-formulas’
• Interpret these relations	
• Discover a new type of symbolism	
• See the T/R strategy and make it run on a very simple exercise	
2. Working on formulas and on variables	
Transition from what is numeric (digital) or verbal towards what is symbolic	
Transition from what is specific towards what is general	
• Approach the generality through numerical calculations issued from the same formula	• Play on the relation between two columns
• Find and write a formula that ...	• Use the functionality of recopy
• Use algebraic transformations to explain the equality between two sets of results obtained by two different formulas	• Spreadsheet objects: ‘columns-formulas’ and ‘column- variables’
Manipulating formulas, approaching the notion of variable through substitutions	
• Numerical substitutions in expressions	• Play on consecutive relations between several cells
• A variable of an expression is substituted by another expression	• Spreadsheet objects: ‘cell-variables’
• Operate on variables to find an equivalent expression or to ‘undo’ a given formula	• Play on the ambiguity of cell reference, a formula container but also a variable for another formula.
3. Approaching the resolution of algebraic problems	
Transition from a work with what is known towards a work with unknowns	
Transition from the application of intuitive methods (not algebraic) towards the application of (algebraic) rigorous school methods	

TABLE I

Continued

Algebraic knowledge	Spreadsheet functionalities
• Find the intermediary expressions corresponding to intermediary equations of a more complex algebraic problem	• Organise a sheet of calculation
• Find unknowns of the problem by the T/R method	• Use the trial and refinement strategy in the spreadsheet environment

4.2. *Pupils' Functioning with Spreadsheets*

4.2.1. *The Surprising Session 1*

The first spreadsheet session was composed of three parts whose goal was to have pupils gain some familiarity with spreadsheets (specific vocabulary, locations on the sheet); discover formulas and the 'handle' of re-copy. For this session, comparing the two classes was very interesting as there were evident common points (same objectives, same organisation in homogeneous pairs, same content, similar teacher practices...) but also some remarkable differences. In the following, we focus on the three main ones.

- The pupils' profile.

  Class A consisted of studious pupils coming from social upper/middle classes and without difficulties in mathematics, whereas class B consisted of pupils having generally many problems in their families and many troubles at school (difficulties in understanding, writing, and with discipline and authority in general). In fact, pupils who did not found a place elsewhere, or had been excluded, were gathered in this class in order to create the single grade 7 of a brand-new school. Since the beginning of the year, the daily life of this class was often punctuated by fights, serious insults (including towards the establishment staff), or thefts. In the course of the academic year, newcomers were regularly arriving because they had been excluded from other schools. About two third of this class were low achievers in mathematics.
- The spreadsheet pre-requisites.

  In the same way, the teacher of technology[6] had initiated the pupils of class A into using spreadsheets during their first term. Those of class B had never been offered such an initiation neither to spreadsheets, nor to computers (when the experiment took place, this new school had just received some computer equipment) so that one of the pupils had even never seen a 'mouse'.

- The orchestrations of session 1.

Lastly, the orchestration of the two classes were different: Dan had got, in her computer room, a video-projector and a computer at her desk, which provided her the possibility of quickly introducing vocabulary and main functionalities (edit and correct a formula, recopy a formula...) by projecting on the board the spreadsheet screen of her computer. In my case, there was neither video projector nor computer at the desk, just a white board[7]. Thus, the didactic configurations were different and their exploitation modes were different too. Dan planned an orchestration in two stages: explanations of the teacher with the use of the video-projector then pupils' work in pairs, whereas I planned a three-stage configuration: reading the instructions and collective answers, then explanations by the teacher using the whiteboard and finally pupils' work in pairs.

- Dan's previsions.

By taking into account these differences, it seemed reasonable to foresee distinct dynamics in the two classes. This is why, when finalising the preparation of her session, Dan thought that the tasks we had designed would certainly be easy for her pupils and quickly solved (also because there were less stages in her orchestration), and that too much time was allocated to them in the script. She also consulted the professor of technology who confirmed that the pupils had already seen the content of this session and that they would certainly end within half an hour. Therefore, Dan decided to include the planned second session in the same hour and prepared pupils' work for that. Let us see what happened.

- What resulted?

The results were very surprising because they showed great similarities between the two classes concerning the answers (types of oral or written answers, errors), the pupils' difficulties (with cells, formulas, functionality of copying down) and, most of all, the time! Actually, this first lesson lasted a full hour for most of Dan's pupils; the same as in class B!

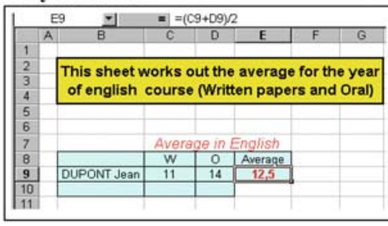
Here are, for example, the details of the the three tasks of the third part (Table II).

The results, in Table III, show some unexpected similarities between the two classes.

We observe quite the same success rate for the spreadsheet manipulation (try and find), and (which is more surprising) for the

TABLE II

A Try and Find



Try and find…

If Jean has 17 for his Oral mark,

1. What would be the average (put 17 in D9) ?
2. What should be his Written mark in order to average 15 (try numbers by modifying C9) ?
3. What is the best average he may reach?
4. What is the lowest average he may reach?

B Predictions...

1. In your opinion, which formula is hidden behind the cell that calculates the General Average?

   Make a prediction:

   The formula in E9 =

2. Observe this formula now (click once above and look at the bar of formula)?

   Which cells are used in this formula?

C Calculate your average...

Under the notes of Jean write your own English notes and calculate your average by copying down the formula of E9 with:

The handle of recopy : click on the cell to be recopied (E9), catch the small black cross with the left click and drag towards E10. (keep the finger on the click while dragging)

Which formula is contained in E10?

TABLE III

	Class A (24 pupils)		Class B (28 pupils)	
	Correct	Wrong	Correct	Wrong
A Try and Find	15	9	17	11
Success	62.5%		61%	
B Predictions…	13	11	7	21
Success	54%		25%	
C Calculate your average	8	16	9	19
Success in using the handle of recopy	33.5%		32%	

use of the handle of recopy: *in both classes*, only about one third of the pupils succeeded in using this functionality. Of course, there are also differences, see for instance the results concerning the prediction of a formula, which certainly reflect the differences in mathematical achievement of the two classes. Another difference is that pupils who did not have enough time to finish the last question were fewer in class A. Nevertheless, session 2, which Dan included, was not started in this first hour with class A and this was completely unexpected by Dan. How can we explain such similarities? In the talk, Dan said:

> I am very disappointed by seeing the results of the pupils especially after what the teacher of technology had said to me: 'they had already seen these contents', 'that would be done by half an hour' etc. And during my collective presentation they did seem to understand. In fact, there are still basic things, which are not acquired, they have forgotten or I don't know.

In our opinion, in order to understand these phenomena we would have to better know the kind of instrumental genesis that had taken place in technology lessons and the mathematical organisations that had supported it, in their different components. Were they really the same as those at stake in this session as the teacher of technology said at seeing the tasks? And, even if so, which instrumentation does it remain one term after? Whatever had been achieved in the technology sessions, was that enough to ensure an instrumental genesis able to resist a one-term break? Research carried out within the frame of the instrumental approach has evidenced the complexity of instrumental geneses and showed that they are necessarily long-term processes. The unexpected similarities we observed in this first session can result once again from the poor sensitivity of the educational systems to these characteristics. We can hypothesise that the lack of practice during one term had serious repercussions on pupils' performances. Similar results can be found in a research reported on a web site[8] calculators: the most effective pupils are those which have their tool, and can make of it use beyond the school hours''.

Concerning the difficulties met by her pupils with the functionality of recopy, Dan was also very disappointed:

> I showed the recopy, I orally explained it, I explained again and again but in fact, till the Session 4, it was still not ok! (...) But orally, that does not work. Even with the video, I think the best is to have a classroom with the computers and tell them 'now, go on spreadsheets for 5 minutes and do it'. Thankfully my classroom is near the computer room but this one is not necessary available at the moment needed, the same with the video projector: you have to reserve it, to foresee everything in advance, this can be done, but you really must be very well organised!

This extract leads us to the second point.

4.3. *Supporting Instrumental Geneses: A Complex Task for Teachers*

Beyond the lack of material or the technical difficulties one could encounter in a computer session, building a mathematical progression involving spreadsheet is a rather complex task for the teacher. It is not only a question of organising a sequence of mathematical objectives but also of managing, according to the instrumental approach, the use of the spreadsheet and its impacts on the expected learning. As stressed in the theoretical part, from a teaching point of view, integrating a tool requires that the teacher simultaneously takes into account different dimensions: the tool's features, the instrumented techniques and 'the concepts involved'. In our experiment, this complexity was regularly brought to light and we would like to illustrate this point by some significant episodes.

- A 'vocabulary' problem arising from the consideration of spreadsheet knowledge. For example, I wanted the pupils to keep trace in their notebooks of the algebra and spreadsheet knowledge at stake in the first session, and organised for that purpose a moment of institutionalisation. Some difficulties linked to the vocabulary arose then. While speaking of a cell, I wrote "a cell has an address which is shown in the 'address zone' of the tool bar, a content which is shown in the 'formula zone' of the tool bar and a feedback shown on the screen in the cell itself". But I felt confused by the two levels of this feedback: a temporary one (formula) and a permanent one (numerical result of the formula), the problem being that both are contents! I immediately thought "I should have said: a *permanent* apparent content and a *temporary* apparent content", but I realised that this last one is also permanently apparent in the tool bar when clicking on the cell, and *even permanently apparent in the cell* when double-clicking on it! So I thought about another formulation, which still did not satisfy me and so on... This incident disturbed the lesson, and made me personally feel the difficulties teachers can experience when they have to invent ways of institutionalising some spreadsheet knowledge!

This vocabulary problem was not an isolated case. Here is an extract of Dan's interview:

Researcher: Did you want to have a synthesis of the work with pupils after the Session 1?

Dan: Not before the session, but looking at the groups, then, I said to myself it would be good to return on the very basics. When I explained "Cell", "Line", "Column"... I was at the video projector and I had the impression that they understood but when I went to see each group I thought I need a "check up" (...) *focused* on the spreadsheet. (...) with Geoplan,[9] it is not so... hum..., here they write formulas whereas with Geoplan, generally everything is already present: if they want to draw a perpendicular bisector, they go to the command perpendicular bisector, *it is the same. It is may be more easy to get in it than to get in a spreadsheet.*

In this extract we also see a reference to dynamic geometry software, this relates to the second point we would like to evoke.

- Managing these two levels is more or less easy according to the tool at stake. Along this experiment, as a teacher, I automatically made comparisons with my use of Cabri-géomètre. I had the feeling that I had experienced it with much more ease, as if its integration in my teaching practice was more natural, as if the tool was more "transparent" or its instrumental processes[10] closer to the traditional paper pencil one. As shown in the extract below, Dan expressed the same idea about her experience with another geometry software:

With Geoplan, it is the same process than spreadsheet: we see things, we observe and then, we go back in the classroom and we demonstrate them with mathematics. But, Geoplan is geometry and, in a certain way, obligatory because pupils must do the same with paper–pencil. With Geoplan, we save time, pupils use it quite quickly and the software helps the weakest pupils whereas spreadsheet hum... we can learn without spreadsheet and manage with the only paper and pencil.

In the following of the interview, Dan is expressing some of her difficulties concerning the experimentation:

Researcher: whereas spreadsheets are not necessary a help for those pupils?

Dan: No, indeed. (...) but in all manners, one always needs a startup in anyway (...) When we worked on relative numbers, I wanted to use spreadsheet to calculate distances between points. But, in this case, there is a condition to put in the formula, I said myself "I won't do it, I will loose my time". But I should have done it! If I were sure having the same pupils next year, I'm sure I would keep on working with spreadsheets. But if I need to re initiate them, I won't do it for all my classes, just for one class, which I would chose in the beginning of the year, but not for all of them. (...) The difficulty is always the same: teachers don't work together, it's very difficult, each year, to have a class and to educate them again, it is exhausting. So, we go fast, the bright pupils succeed, not necessary those who we precisely wanted to help. And it's not easy to manage a class: half class has already been initiated, other half not. It is not obvious to manage all this.

In this excerpt, Dan raises many interesting problems: the question of the time spent to get familiar with a tool, which seems to depend on the tool, and the disregard of this difficulty by the school system. Dan also highlights the *double teamwork necessary* to undertake spreadsheet integration: between different teachers of a same class (mathematics and technology here) and with those who teach different grades. This relates to the pupils' experience in spreadsheet use, to their level of instrumentation. Finally, Dan evokes some resulting problems about class management and about the pupils' heterogeneity that the tools tend to increase; and here again this seems to depend on the tool.

4.4. *Conclusions: Two New Hypotheses*

In conclusion, this experiment stresses the importance of instrumental geneses and tends to confirm that they are not obvious, that their guidance raises many didactic problems. Undervaluing them can contribute to the difficulty observed in integrating spreadsheets in mathematics education. These results also lead us to formulate new hypotheses to test.

1. A first hypothesis is the following:

> The more complex the instrumental process is, with regard to the traditional referent environment (paper and pencil), that is to say, the bigger is its distance to the 'current school habits', and the more difficult the integration of the tool is.

In the spreadsheet case, the instrumental needs are particularly strong; we gave an example of the 'distance' from the paper and pencil with the notion of variable-cell. However, how can we explain more precisely the important differences felt by Dan at integrating two different environments in her practice? How can we characterise this vague notion of relative complexity of the instrumentation process, of relative distance to a paper pencil environment? This question remains an open problem and our study only begins to answer it.

This distance seems to be more or less significant according to the tool at stake, and thus requires, more or less, the work that Guin and Trouche discuss with regard to CAS:

> Therefore, we argue for strong teacher involvement in the instrumentation process and full recognition of the constraints and potential of the artefact as well as various profiles of students' behaviour so as to design and implement appropriate mathematical activities. Teachers have to juggle all these parameters in order to enhance students' experimental processes of combining information and understanding tools. How should teachers organise their teaching in order to turn symbolic calculators into efficient mathematical instruments? (...) Teachers should consider the instrumentation process in order to articulate new techniques with older practices in the paper/pencil environment, because this reorganisation of instrumented techniques is far from spontaneous and requires spending sufficient time to reach the experimental processes. (Guin and Trouche, 1999)

In the case of mathematics education, this "distance" is presumably related to particular characteristics of the software. We would like, now, to try to clarify these characteristics. For example, some aspects of Cabri (similarly with other software following the micro worlds tradition) are carefully *designed with students' learning in mind*, whereas spreadsheets were not created for mathematics learning at all. Can this be such a characteristic?

Another one could be *the vocabulary involved in the tool*. In Cabri, even if it is not totally the same, the vocabulary is quite close to the usual one in geometry (points, lines, circles, symmetry...), whereas the vocabulary in spreadsheets is far from the mathematical one, the user must even *create* it by him/herself, as we have seen it in the experiment. There is no official reference to help him/her and, above all, when the user is a mathematics teacher, she/he also has to relate this vocabulary (and spreadsheets objects) to the mathematical ones (what is a cell: a variable? What is a column (or a line): several variables? Or another representation of a unique variable? What is a relative address? Is there an algebraic equivalent? What is 'filling down': a formula? Is the numerical feedback a number or a result of a formula? Or is it the permanent appearance of the cell containing a formula whereas the formula itself would be its temporary appearance?...)

Finally, a last characteristic could be linked to the didactical and the epistemological status of the tool. According to Chevallard (1992), any technical object brought into teaching is not neutral: it changes the knowledge taught since it entails actions on this knowledge. These actions must thus appear as legitimate manipulations in the didactical system. Traditional objects progressively built their didactical legitimacy, what about new tools? Teachers without any problem integrate ruler and compass, they are part of the mathematical culture. Is it because, historically, they played an essential and epistemological role in the development of mathematics? (Chevallard, 1992) This role and the number of mathematical problems they generate legitimate their place in mathematics education. Is it the same for spreadsheets? How is their introduction in mathematics teaching justified? Do teachers feel this tool relevant to *their* mathematics and the ways they learnt, do and teach mathematics?

2. A second hypothesis concerns the various resources, which, as we have seen in another part of our research, do not take into account (or do so very rarely) instrumental genesis in the development of mathematical concepts. Only the mathematical aspect of pupils' work is explicitly treated, but the rest remains implicit (which relates to the implicit elements shown in the theoretical part, Section): there is no element helping the teacher manage instrumented activities (at what moment does it take place? what are its spreadsheet prerequisites?...). As if this setting up was obvious, which is, as we saw above in the experiment, completely false; on the contrary, it requires important work and deep reflection from the teacher. We put forward this second hypothesis, which relates to the role of the teacher stressed above.

> A teacher who is a "non expert" of the tool is poorly sensitised to the tool's potentialities. First, she/he sees some differences/added complexity, she/he is poorly prepared to combine instrumentation and mathematics learning and, for these reasons, she/he hardly gets any benefit from current resources.

Our present research aims at testing these two hypotheses by:

- clarifying the characteristics of the complexity of an instrument with regard to its integration into mathematics teaching;
- studying how teachers integrate the spreadsheet: by analysing the practices of experts using spreadsheet (teachers and teachers trainers): do they pay attention to the differences of objects, of techniques between paper–pencil and spreadsheet environments? Do these aspects play an important role in the integration of

spreadsheets into teaching or are there only the reasons usually evoked: teachers' fear of being inefficient, lack of effort, resistance to changes in practices, material problems, lack of training...?

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NOTES

1 Computer Algebra System.
2 Even if spreadsheet does generate one.
3 To avoid confusions, we did not mention the component 'technology' which designates the discourse about the techniques (explanations, justifications...) as we mainly use this word with another sense in this text.
4 We limit ourselves, in this brief synthesis, to the case of the material artefacts, but the ergonomic approach is extended to 'psychological' artefacts: symbols, signs, cards...
5 Because of this dialectics "it is not possible to clearly distinguish between these two processes" (Trouche, 2003a).
6 In France, among the various disciplines taught in middle school, there is one named "Technology" where pupils learn the usage of various technological tools (computers, software as word processors, spreadsheets...).
7 White board, without squaring which could have accelerated the representation of worksheet or cells.
8 http://www.univ-reims.fr/URCA/IREM/tableur/conclusions.htm
9 A dynamic software for geometry created in France especially for education.
10 That is the processes at stake in instrumental genesis: instrumentalisation and instrumentation, see Section 3.1.

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