Computer algebra, instrumentation and the anthropological approach

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Introduction

My brief for this paper was to continue the CAME debate on "French CAS work[i]". I will give an "outsiders" impression of this work and raise issues for discussion by my reactor and at this CAME Symposium. I hold this work in high regard. It has provided me with insights but I have often found it "difficult." I will try to convey these insights and difficulties in this paper. Through reading and discussion I have found that I often interpreted ideas and terms in ways that the authors did not intend. I raise some of these differences in this paper when I think they are worthy of discussion. I am an Englishman writing of French work to a largely North American audience. I attempt to refer to North American authors whenever possible.

Many of the ideas developed by French CAS workers, as well as ideas that inform their work, are complex. It is quite impossible in a moderate length paper to detail them all and the task would probably be beyond my capabilities. I attempt to describe detail that seems fruitful to take up as points for discussion but otherwise refer to the literature. Regarding the structure of this paper I start by considering the historical setting from which French CAS work arose. I then devote a short section to tools, as how we view tools is a non-trivial matter. The main section outlines the theoretic basis for French CAS work: the so called ergonomic approach (instrumentation) and the anthropological approach[ii]. In the final Discussion section, I address ambiguities/tensions in these two approaches, ask whether it is possible to integrate these two approaches, attend to missing elements, examine whether it may be fruitful to relate French CAS work to the work of others,and finally look towards how this work may help us improve the learning and teaching of mathematics.

Historical considerations

In the early 1990s there was a burgeoning of mathematics teachers and education researchers in the developed world conducting innovative work in high school and college mathematics classrooms with computer algebra systems (CAS). My impression was that the countries most involved in this work were Austria, France, the USA, and the UK and that many people felt that CAS had the potential to free students from manipulations and allow them to focus on concepts and complex problem solving (see Pozzi, 1994). Austria and France stood out, in my opinion[iii], as having some organisational unity surrounding individual studies. Many of the individual projects in both countries were sponsored and/or supported by their respective Ministries of Education (see Heugl, 1996; Hirlimann, 1996). Apart from teachers and Ministry workers, CAS groups in both countries enlisted the support of others. Austria had, and still has, the impressive RISC Institute at the University of Linz, led by Bruno Buchberger who was responsible for groundbreaking work in symbolic computation. Buchberger had a number of educational ideas, the most well known being his "white box – black box" and "black box – white box" principles about when a CAS should and should not be used (see Drijvers, 1995; Heugl, 1996). CAS projects supported by the Austrian Ministry adopted these principles and two further principles: the "window-shuttle" principle which concerned multiple representations and the "module" principle which concerned white (black) boxes created by (used by) students. The final outcome of these influences on Austrian CAS work, in my opinion, was a response to "how can we use these principles to improve the learning of mathematics?"

In contrast the French Ministry CAS group enlisted the help of a mathematics education research team led by Michèle Artigue. This led to research questions concerning actual use of CAS (as opposed to desired use), to identification of difficulties and obstacles encountered by students and teachers, and to emergent principles (Hirlimann, 1996). Research work found "that the occasional use of a CAS ... adds some technical difficulties to mathematical problems that may impede students' progress in the mathematical activity itself" (Hirlimann, 1996, p.4). A number of key researchers were concerned about their interpretation of the phenomena they observed. Speaking about this period (the mid-1990s) Artigue writes:

> ... we needed other frameworks in order to overcome some research traps that we were more and more sensitive to, the first one being what we called the "technical-conceptual cut". Indeed, theoretical approaches used at that time ... tended to use this reference to constructivism in order to caution in some sense the technical-conceptual cut, and we felt the need to take some distance from these. Artigue (2002, p.247)

At this time they were influenced by the work of Vérillon and Rabardel (their 1995 paper is the most common reference to their work) on instrumentation (the so-called ergonomic approach) and Chevallard's anthropological approach. In the anthropological approach, "praxeologies" (practices) are described in terms of tasks, techniques (used to solve tasks), technology (or "talk", the discourse used to explain and justify techniques), and theory (see Chevallard, 1999). The conjoining of these two approaches was, at one level and in my opinion, an act of brilliance but, at another level, accounts for tensions in the corpus of the various studies and papers. I consider some of these tensions later in this paper.

On tools

A large part of this paper focuses on tool use. Followers of both the ergonomic and the anthropological approaches are likely to object to my use of the word "tool" so I take a short but important tangent on what the word means to me.

I distinguish between an artifact and a tool. An artifact is an object that has, or had, an intended purpose, but an agent need not be aware of this purpose or may appropriate it for a purpose not originally intended (e.g. using a calculator as a straight edge). An artifact is material. In the case of the calculator this is obvious – we can touch it. But, less obviously, material artifacts such as algorithms are created and used in mathematics. The materiality of an algorithm is less immediate than the materiality of a calculator but it nevertheless exists in the materiality of its spoken or written form (without a sign form it cannot exist). The fact that any algorithm can be programmed into a computer attests to its materiality. A tool, to me, is an artifact whose purpose is to perform a task or set of tasks, and, as an artifact, it is material.

I encounter phrases such as "maths is a tool" and "maths is a tool-box" used interchangeably in students' essays. I usually do not take issue with this unless they are specifically writing about tools since I understand what they mean to say, but I always think, "In what sense is it a tool/tool-box?" In academic works it is common to find reference to "semiotic tools" and "cultural tools" (see, for example, Werstch (1998)). These terms are, to me, generally clearly defined in material I read though I do like the adjectives to be included; all tools are ultimately based on human interaction with the material world but I believe we interact with cultural, physical[iv], and semiotic tools in different ways. Although I can confuse myself, sometimes I attempt to use the word "tool" on its own when it takes on several forms; for example, a calculator is a cultural, a physical, and a semiotic tool.

Tools are omnipresent, so it is easy to overlook them! Yackel and Cobb (1996), for instance, focus on verbal interactions in the formation of socio-mathematical norms but, as Herschowitz and Schwarz (1999) point out, "... socio-mathematical norms do not arise from verbal actions only, but also from computer manipulations as communicative non-verbal actions." Tools, and more generally artifacts, permeate every aspect of our lives to the extent that we cannot account for our "being" without taking them into account.

Tools have always been very important things. Gibson (1993), a biological anthropologist, argues that the interdependence of tool use, language, social behavior, and mathematical concepts (information processing capacities) distinguishes humans from socially functioning, tool-using apes. In education, mathematical tools "shape and are shaped by the actions of the students" (Hoyles, 2003, p.2), but how we view such a statement is bound up with our interest or viewpoint. I now want to briefly consider this transformational feature of tools and how we view this transformational feature.

Tools have always transformed actions but this has not always been stressed. Wertsch (1998) documents how, in pole vaulting, the introduction of the fibreglass pole, instead of an aluminum pole, in the 1960s transformed, by "flinging" the vaulter, the actions of pole vaulting. Noss and Hoyles (1996) consider constructing the midpoint of a line segment by a compass, by measuring and by *Cabri*; the mathematical actions are totally different in each case. Similarly, using CAS to differentiate $f(x) = x \sin x$ involves a totally different set of mathematical actions than is required to differentiate this function using the standard paper-and-pencil algorithm. Of course, anyone with the training to do the differentiation with both tools realises this is different; the point is whether or not they say, in so many words, "Hey, this difference is really important! What does it signify?" Wertsch (1998) makes the point that we can understand the transformational power of tools by considering different *genetic domains*: phylogenesis (such as Gibson does); sociocultural history (e.g. the appearance of digital technology); ontogenesis; or microgenesis (moment-by-moment development in social interaction; this is what CAS researchers invariably focus on).

The instrumental approachand the anthropological approach

The instrumental approach

CAS is a tool, a very complex tool which incorporates various computational media (Cuoco, 2002). In their quest to understand CAS-as-a-tool French CAS researchers turned to French research on *instrumented activity*. Vérillon and Rabardel (1995) distinguish between a tool, as a material object, and an instrument as a psychological construct: "the instrument does not exist in itself, it becomes an instrument when the subject has been able to appropriate it for himself and has integrated it with his activity."

Trouche (2003) provides a clear exposition of the psychological foundations of the instrumental approach. He considers an instrument as an extension of the body "made up of a tool component (a tool, or a fraction of a tool mobilized in the activity) and a psychological component." He emphasises the subject-tool dialectic and names the direction of the influences: *instrumentation*, how the tool shapes the actions of the tool-using subject; and *instrumentalisation*, the ways the subject uses (shapes) the tool. The psychological component is explained via the Piagetian notion of a *scheme* "the structure or organization of actions as they are transferred or generalized by repetition in similar or analogous circumstances" (Piaget & Inhelder, 1969, p.4). A scheme, Trouche argues, has three functions: pragmatic, it allows the agent to do something; heuristic, it allows the agent to anticipate and plan actions;

and epistemic, it allows the agent to understand something. The evolution of this dialectic between tool and scheme is called "instrumental genesis." This is a complex process over time which links the affordances and constraints of the tool to the agent's prior understandings and activity. Trouche introduces the notion of "gestures." Gestures are observable behaviours and a "scheme is the psychological locus of the dialectic relationship between gestures and operative invariants, i.e. between activity and thought." Trouche's use of schemes and his notion of gestures are, I believe, his way of relating internal and external phenomena (I return to this point in the Discussion section). Other constructs introduced by Trouche are techniques and instrumented techniques (see Trouche, 2005). Techniques are sets of gestures realized in the execution of a task on an instrument; instrumented techniques are techniques involving artifacts and are thus the gestures of instrumented action schemes.

There are, I feel, a number of theoretical issues to resolve with regard to these constructs and I raise several of these issues in the Discussion section. What is clear, however, from a practical mathematics educators' perspective is the immense interest of, and insights provided by, French CAS workers' analyses of student-generated schemes and students' use of techniques. These analyses have, moreover, helped many to see that which, perhaps, should have been obvious, that CAS as they are now designed present the user with everything s/he can do and thus constrain the user's potential to develop his or her own symbolism (this being especially true when the user is a student). Given this, then maybe theoretical issues are relatively unimportant for practical mathematics educators. I will not return to this in this Discussion section but it may be something we wish to discuss.

Another aspect of French CAS work arising from instrumentation considerations is *orchestration*. I did not intend to introduce this theme in my presentation because it was a focus of the last CAME Symposium and because I felt I was already covering a lot of material. It is, however, relevant to issues I raise in the Discussion section. I keep my summary brief. Trouche (2004, p.296) states, "I introduce the term *instrumental orchestration* to point out the necessity (for a given institution – a teacher in his/her class, for example) of *external steering* of students' instrumental genesis." Guin and Trouche (1999) introduced the term "orchestration," in this context, to English-reading audiences. It is a study of a particular technological classroom environment that includes students with TI-92s and exercise books, a rotating (amongst the class) "sherpa" student who operates the viewscreen, a viewscreen, a blackboard, specific tasks, and a teacher. They note the instrumental genesis of students through an examination of change in student mathematical behaviour and in student-tool trajectories throughout the instrumental process. In commenting on how to support instrumental genesis they argue for strong teacher involvement in the instrumental process and recognition of the constraints and potential of the artifacts and of student behaviour. It is not, in my opinion, a study of teachers but, rather, a construct-rich account of how one rather excellent teacher supported instrumental genesis. Trouche (2004) later describes orchestration that takes on a wider interpretation so that sherpa-student-orchestration comes across as one of many possible forms of orchestration.

The anthropological approach

Tool use does not exist in a vacuum, tools are used in a context/in practice/in activity but how one views practice/activity is very important. Many French CAS workers view mathematical activity/practice in terms of Chevallard's (1999) anthropological approach where, as noted above, practices are described in terms of tasks, techniques, technology and theory. These four Ts fall into two sets: technology and theory, which concerns what is legitimised as knowledge *per se*; and task and technique, which concern "know-how" or, rather, "know-how relevant to a particular theory and technology"[v]. In this paper I focus on task and technique.

Task and technique are particularly important in the writings of Artigue and Lagrange. If nothing else, French CAS workers have provided mathematics educators a service by focusing attention on the form and role of tasks and techniques, particularly with regard to their solution with the aid of digital tools. Before considering tasks and techniques further it should be noted that tasks and techniques are considered by these writers in a somewhat different way to the way they are viewed in much mathematics education literature: tasks are artifacts that are constructed (and reconstructed) in educational settings (institutions[vi]); techniques are not simply technical manipulations but are institutionally privileged to the extent that only one, of many possible techniques, may be considered. In contrast to this view of tasks it appears that researchers who consider tasks outside of the anthropological approach focus on qualities of tasks that facilitate specific mathematical actions or observations. Sahlberg and Berry (2003), for example, focus on the qualities of tasks that do and do not facilitate small group discussion and Monaghan and Ozmantar (in press) devote a section of their paper to considerations of the qualities of tasks that facilitate the consolidation of an abstraction.

There are two aspects of tasks and techniques which can be overlooked but are important: (i) there is nothing natural about specific tasks or techniques (there are many things we can do and, invariably, many ways to do them); (ii) social values are attributed to specific tasks and techniques. Returning to differentiating $f(x) = x \sin x$, I wonder why we ask students to do this and why we ask them to do this in a particular way. There are many ways to answer these questions: that certain tasks are important because they are generic in the sense that once you can solve one of a family of tasks, you can solve the others in that family of tasks; that certain tasks are important to set because solving them using a particular technique is likely to lead to a desired understanding; that certain techniques are mathematically efficient and/or elegant; and so on. There are values behind such answers. Artigue (2002) and Lagrange (2005_b), amongst others, differentiate between pragmatic and epistemic values of techniques. Pragmatic values concern the efficiency, or breadth of application, of a technique. Epistemic values concern the role of techniques in facilitating mathematical understanding. I regard these constructs as very important and they are things I did not explicitly consider until French authors introduced them to me. Take, as Lagrange (2005_b) does, expressions of the form $\frac{a+b\sqrt{2}}{c+d\sqrt{2}}$, where a, b, c and d are integers. The standard technique of multiplying top and bottom by $c - d\sqrt{2}$ has pragmatic value in writing any such expression in the form $\alpha + \beta\sqrt{2}$ and potential epistemic value in developing students' knowledge of properties of quotients and radicals. Artigue extends these ideas:

> Professional worlds as well as society at large have a pragmatic relationship with computational tools: their legitimacy is mainly linked to their efficiency. But what schools aim for ... is much more than developing an effective instrumented mathematical practice. The educational legitimacy of tools for mathematical work has thus both epistemic and pragmatic sources: *tools must be helpful for producing results but their use must also support and promote mathematical learning and understanding*. (Artigue, 2005, p.232)

The aspect of Artigue and Lagrange's work that has caused the most controversy is their claim that the relationship between techniques and conceptual understanding is a highly complex one (or, to put it bluntly, that using technology to bypass techniques is not a quick way to conceptual thinking). The argument for this (see, for example, Lagrange (1999)) is essentially in five parts:

1. Technical work does not disappear when doing mathematics with CAS, it is transformed.
2. Within a theory, every topic has an accompanying set of tasks and techniques. Novices progressively become skilled in techniques by doing, talking about, and seeing the limits of techniques. This eventually leads to a theoretical understanding of the topic.
3. Although rote repetition of a specific technique for a specific task is a mathematically impoverishing experience, this is not a reason to jettison techniques *per se*.
4. Techniques and schemes are linked. Students need time to develop rich schemes by using techniques.
5. The empirical observation that diminishing the role of techniques encourages teachers to avoid talking about them (Chevallard's "technology").

These considerations must be taken seriously by practical mathematics educators concerned with the introduction of technology into mathematics education; a consequence of ignoring them is that we attempt to implement ideals without realising conflicts in practice. I do not return to this in the Discussion section but I expect that it is something that we will want to discuss.

Discussion

French CAS work is, in my opinion, of undoubted importance. It is, however, ongoing research and scholarship and further work is needed:

- to clarify ambiguities,
- to integrate (if possible) the two main approaches upon which the work draws,
- to attend to missing elements,
- to relate it to the work of others, and
- to apply it in efforts to improve the learning and teaching of mathematics.

To this end I am deliberately critical in this section and raise matters that CAME delegates may, if they wish, take to the discussion forum at the Symposium. I structure this section around the five bullet points above.

An ambiguity/tension

A tension, as I have previously suggested, comes from differences of emphasis and of focus between the ergonomic and the anthropological approaches: the former looks inward to the agent-tool dialectic, the latter looks outward to the social setting the tool using agent finds herself in. A particular instance of this tension is the meaning of the term "technique" in works by Lagrange and Trouche. Trouche (2005a) focuses on the agent-tool and the psychological construct of a scheme,:

> One can describe human activity (and students' activity in particular) in terms of *techniques*, i.e. sets of gestures realized by a subject in order to perform a given task. When a technique integrates one or several artifacts, we will speak of an *instrumented technique*. Instrumented technique is thus the observable part of an instrumented action scheme.(p. 151).

Lagrange (2005b) uses Chevallard's sense of "technique," a way of doing tasks:

> Techniques help to distinguish and reorganize tasks. For instance different techniques exist for the task "find the intervals of growth of a given function" depending on what is known about the function. If the function is differentiable the task can then be

> related to the task "find the zeroes" of another function. In other cases, a search based on a more direct algebraic treatment can be more effective. (p.116)

I think it must be questioned whether these two authors are talking about precisely the same thing when they use the term "technique." Perhaps the authors are focusing on different genetic domains with regard to tasks: Trouche on microgenesis and Lagrange on sociocultural history.

Is it possible to integrate the two main approaches?

I have a strong interest in trying to "see the whole picture," developing a holistic analysis, so I am, perhaps, seeking a unity between these approaches that is not desirable. Lagrange has said (private correspondence) "It seems that the best thing is not to try to 'merge' these two approaches, but rather to emphasize their specific contribution." These problems are not peculiar to French work. In the United States a similar argument goes on. Daniels (2001, pp.78-79) considers Wertsch as advancing the case for the use of mediated action (the agent-tool dialectic) as a unit of analysis of sociocultural research and contrasts this with Engeström who points out the danger of under-theorising context. Cole tries to develop a middle line but,

> Mediated action and its activity contexts are two moments of a single process ... It is possible to argue how best to parse their contribution in individual cases, in practice, but attempting such a parse "in general" results in empty abstractions, unconstrained by the circumstance. (Cole, 1996, p.334).

Nevertheless, an integrated approach would be an advance, and Artigue (2005) shows that rich interpretations of teaching and learning experiments which draw on both approaches can be made. It is possible that Vergnaud's Piagetian concept of schemes is a stumbling block to integration. I return to this matter, after discussing tensions between Piagetian and Vygotskian perspectives, in the subsection *Links with other approaches*.

Missing elements

French CAS workers have addressed a number of important areas in the teaching and learning of mathematics with technology, so it is a bit unfair to focus on missing elements. There are, however, two important areas which, if considered, would make the corpus of their work more complete – teachers and affect.

Regarding teachers, it must be noted that teachers are not ignored in French CAS studies, they are simply not, to my knowledge, the focus of their studies. This need not be a criticism if a holistic approach had been developed, for then teachers would simply be a part of the picture. My belief that teachers have not intrinsically featured in the work of French CAS authors seems to be borne out by the fact that the chapter focusing on teachers in the recent book on French CAS work (Guin et al., 2005) was written by three Australians (Kendal et al., 2005) who do not adopt either the ergonomic or the anthropological approach. It could be argued that orchestration is an instrumental approach that examines the role of the teacher in facilitating students' instrumental genesis. While this is true at one level (it is an instrumental approach and teachers do feature in it) it is, I feel, more an account of what can be done than it is a study of teachers' practices. I question too whether the class' influence on the teachers' actions and the co-construction of socio-mathematical norms are accounted for in this model.

I feel a more grounded approach is need. It would be useful to conduct studies focused on teachers to develop accounts of their instrumental geneses, which are surely likely to differ from their students' instrumental geneses because of their mathematical histories. Such studies should, I believe, also focus on the *instrumentalisation* of CAS by teachers as they will, undoubtedly, use the tool in ways not envisioned by designers.

Affect (students' and teachers' beliefs, attitudes, emotions and motivations) is clearly important in the process of turning an artifact into a mathematical instrument but appears, to me, to be underplayed in French CAS work. The reason, I suppose, is that affect is not a major component in either the ergonomic or anthropological approaches. This is a pity as affect is clearly very important: the schemes developed by students with positive and negative attitudes to their symbolic calculators are likely to evolve in different ways; values attached to techniques by teachers who have differing beliefs about what (school) mathematics should be, are likely to conflict. I am not calling for a separate study on affect – to separate affect from cognition and practice would marginalise it – but that a more complete analysis would result if affect were more central to French CAS authors' considerations.

Links with other approaches

Studies which stress, as French CAS work does, the importance of context, social activity, and tool use are often lumped together under the term "sociocultural." Should an attempt be made to somehow merge this hybrid ergonomic/anthropological approach with sociocultural approaches? I think not because different approaches have different emphases and different historical roots, for example, Vygotsky's work for activity theory and Brousseau's work for Chevallard. Proponents of different approaches, however, can learn from each other. In the following, I briefly consider potential links and tensions between the theoretical basis of French CAS work and Vygotskian perspectives. I enter the "Piaget vs Vygotsky" debate in this subsection. The logic of the issues I consider force me into this debate and I enter it somewhat reluctantly: although I am strongly influenced by Vygotskian ideas[vii] I do not dismiss all Piagetian ideas. Further, I am a mathematics educator, not a psychologist.

The instrumental approach develops its underlying theory through Piaget's notion of schemes. Piaget and Vygotsky are often placed as incompatible theorists, sometimes on the basis that Piaget views the individual as primary whereas Vygotsky views social life as primary. In mathematics education circles Lerman (1996) argues this line. Cole and Wertsch (1996), two notable Vygotsky scholars, argue that this is too narrow a view and note that "Piaget did not deny the co-equal role of the social world in the construction of knowledge" (p.1) and that Vygotsky "insisted on the centrality of the active construction of knowledge" (p.1).

The important contribution of Vygotsky, to me and to others (e.g. Cole & Wertsch (1996)[viii]), is that the relationship between human agent and the objects of its environment is mediated by cultural means, with tools and signs being particularly important mediational means in the case of mathematics. The oft cited paradigm divide between socio-cultural and cognitive approaches in mathematics education hinges on the centrality of mediational means in mathematical actions. Vygotsky transformed psychology by making mediational means central to all considerations and adopting the method of dialectical materialism where tool and agent act on each other. This tool-agent interaction, however, is basic in instrumentation work: *instrumentalisation* and *instrumentation*. There is commonality here, so where's the difference? The difference, I believe, centers on the following. Although a cognitive perspective does view that artifacts can facilitate mental processes, thinking ultimately reduces to nature and biological adaptive processes. A Vygotskian perspective, however, insists that artifacts fundamentally shape and transform mental processes; almost everything we do is unnatural and mediated by artifacts. This is a significant difference, but it does not, I believe, exclude a meeting of minds on certain issues. I have, for years, interpreted French

CAS work from a Vygotskian perspective but have communicated well with French CAS workers. Interestingly my concerns, from the outset (see Monaghan, 2001), have been with their use of schemes, to which I now turn.

My own interpretation is that when French CAS workers were first trying to make sense of their early observations, they were semi-independently interested in the work of Vérillon and Rabardel and that of Chevallard. They integrated these into their interpretation of data without realising the full implications of adopting these theories, and they were, as a result, "stuck with" these schemes. Now these schemes certainly seem to have validity with regard to application to CAS work by students (they address mathematical actions and repetition). Such schemes, however, would be mental entities and any reasonable theory of mind must, I believe, address how an artifact (external to the agent) becomes an instrument (internal to the agent). Trouche (2005a) explains this by way of gestures: "scheme is the psychological locus of the dialectic relationship between gestures and operative invariants, i.e. between activity and thought." which in turn leads to techniques and instrumented techniques. The constructs introduced have multiplied and there is, to me, a clear need to check the validity of these constructs. I am, of course, asking a great deal as there is no consensus on what "internalisation" is. Vygotsky proposed that "internalisation" is a series of transformations whereby an interpersonal process is transformed into an intrapersonal one, though most agree that he is somewhat vague about details. In addition, the prominent Vygotskian Wertsch (1998) is critical of Vygotsky's account of internalisation, preferring to use "mastery" and "appropriation" in its place. I think French CAS workers should simply cut "schemes" from their vocabulary until the situation is clear. This is especially so in the paper by Drijvers and Gravemeijer [ix] (2005). I regard this work highly but I also regard their instrumented action schemes as techniques (in the sense of Lagrange).

The development of Vygotskian traditions under the name of "activity theory" (e.g. Leont'ev (1978) and Engeström (1987)) may be a way to take the debate forward. Activity theory considers the actions of a person towards an objective (affective motives are thus essential). Activity is mediated through artifacts, social procedures, and language, and Trouche may find the ideas of mediation relevant to his considerations of orchestration. Activity theory in its most comprehensive current form has a great deal to say about the community, about rules, and about the division of labour in collective activity. This aspect is not, by any stretch of the imagination, the same as the anthopological approach but it seems that the two approaches can learn from each other: activity theory can shed light on teacher intention, teacher privileging and student appropriation[x], which would appear to be useful constructs for French CAS workers to address; and activity theory can be enhanced by considerations of tasks, techniques, technology and theory.

Can this work be used to improve the learning and teaching of mathematics?

Improvement of the learning and teaching of mathematics is the touchstone for scholarship and research, or else such work is merely "academic." I would like to note, however, that I do not believe scholarship or research can be applied directly to further research or curriculum development outside of the domain or country in which that research originated; the local situations matter a great deal. All that scholarship and research can do is inform design.

First it should be noted that CAS has a bad name in many places. In France "... CAS are mentioned to warn teachers against misuses of calculators by students rather than to promote a wise use." (Lagrange, 2005c, pp.154-155). CAS calculators are not allowed in A-level examinations in my country and they are not widely used in teaching and learning. Hoyles, Noss and Kent (2004)state that the set of problems leading to the marginalisation of technology "... points, in part, to a failure to theorise adequately the complexity of supporting learners to develop a fluent and effective relationship with technology in the classroom" (p.311). I agree; this is one reason I personally prefer research such as that carried out by French CAS workers to what I called "principle-led" research in the section on *Historical considerations*: we need to understand the problems in classroom practice (for students and for teachers) before we can offer solutions. Ideals are important but idealism is not a solution.

Instrumentation, with its focus on the agent-tool dialectic, and the anthropological approach, especially its focus on the technical-conceptual cut and epistemic/pragmatic values, has, I believe, taken us forward in theorising the complexity of supporting learners but it is not enough in itself. A problem for CAS work is that CAS can appear "as a 'monster' that could do virtually everything" (Monaghan, 2004, p.339). This is surely related to the problem that CAS-in-education workers have paid little attention to design issues, preferring, in general, to work with the design given to them by CAS designers. Recent French CAS work (Lagrange, to appear) has addressed this problem and has developed theoretical and practical rationales for using a CAS kernel in a computing system that directs students towards educationally salient features of high school algebra/calculus problems. This is, I believe, a fruitful line for further research.

Notes

[i] The ideas originated in French mathematics education circles and most of the papers are by French people but it is not exclusively French. I find the term 'French CAS work' useful to describe the whole corpus but I apologise to French researchers who prefer the term 'logiciel' to computer algebra system (CAS).

[ii] I follow French authors in calling these approaches *ergonomic* and *anthropological*.

[iii] Not only in my opinion. The then *The International DERIVE Journal* (now *The International Journal for Technology in Mathematics Education*) had two special issues devoted to national initiatives: Vol. 3, No. 1 on Austrian work; Vol. 3, No. 3 on French work.

[iv] 'Physical' in the sense that it can be touched, e.g. a calculator. This does not prevent it also being, say, a semiotic tool.

[v] This is how I understand it. I usually omit personal comment on Chevallard's approach because: the approach is complex and not easily summarised; Chevallard is held in extremely high regard by French CAS authors who do not appreciate simplistic accounts; and my knowledge of his thinking is meagre. In this paper, however, in which I present a personal view of French CAS work, I think a little interpretation is useful.

[vi] 'Institution' is used by French CAS workers with specific overtones. It relates to Chevallard's notion of 'didactical transposition' where "mathematics in research and in school can be seen as a set of knowledge and practices in transposition between two institutions, the first one aiming at the production of knowledge and the other at its *study*." (Lagrange, 2005_a, 69).

[vii] Vygotsky died a long time ago and a lot has of work has been carried out in his name since then; there is no single Vygotskian perspective.

[viii] Given my comments on Lerman (1996) above, I think it fair to note that Lerman (2001) takes this position.

[ix] These authors are Dutch, not French, but Drijvers works in the tradition of Trouche.

[x] Another word that appears to be used in a different way by French authors and Anglo-American authors. I do not expand on this here but this could be raised in the Symposium discussion.

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