**Name:**

**Activity 1: Equivalence of Expressions**

**Lesson 1**

### Part I (with CAS): Comparing expressions by numerical evaluation

I(A) The table below displays five algebraic expressions and two possible values for *x*.

Using the two given values of *x* (i.e., 1/3 and –5) and two others of your own choosing, calculate the resulting values for each expression by means of the evaluation tool of your calculator [i.e., the “with operator”, (**|**)].

Important: Proceed one complete row at a time when filling in the table.

Record your choice of additional *x* values in the table’s top row, and record the results in the appropriate cells below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| For x = | 1/3 | -5 | 0 | 1 |
| Expression | Result | Result | Result | Result |
| 1. *(x*–*3)(4x*–*3)* | 40/9 | 184 | 9 | -2 |
| 2. *(x2+x*–*20)(3x2+2x*–*1)* | 0 | 0 | 20 | -72 |
| 3. *(3x–1)(x2–x–2)(x+5)* | 0 | 0 | 10 | -24 |
| 4. *(-x+3)2 +x(3x–9)* | 40/9 | 184 | 9 | -2 |
| 5. | 0 | 0 | 10 | -24 |

I(B) Compare the results obtained for the various expressions in the table above. Record what you observe in the box below.

Expression1 = Expression4 for each of the values of *x*

Expression3 = Expression5 for each of the values of *x*

Expression2 = Expression3 only for *x* = 1/3 and *x* = -5

Expression2 = Expression5 only for *x* = 1/3 and *x* = -5

I(C) Reflection question:

Based on your observations with regard to the results in the table above (in (IA)), what do you conjecture would happen if you extended the table to include other values of *x*?

|  |
| --- |
| I would conjecture that my observations hold in general. That is:  (1) Expression1=Expression4 on every common value at which they are evaluated, and (2) likewise for Expression3 and Expression5.  With regard to values of *x* different from those in the table, I would expect Expression2 to yield results different from Expression3 and Expression5 for some of those values and to perhaps coincide for others. |

## Classroom discussion of Part I A, B, C

**Part II (with paper and pencil): Comparing expressions by algebraic manipulation**

II(A) Based on your observations in Part I A and the subsequent classroom discussion, make a conjecture as to which of the above set of given expressions might be re-expressed in a common form?

|  |
| --- |
| Conjectures:  Expression1 and Expression4 are expressible in a common form;  Expression3 and Expression5 are expressible in a common form;  Additional conjectures:  Expression2 and Expression3 are not expressible in a common form;  Expression2 and Expression5 are not expressible in a common form. |

II(B) To test your above conjecture by means of paper and pencil algebra, re-express the given expressions below in another form (not the expanded form). Show all your work in the table’s right-hand column.

|  |  |
| --- | --- |
| **Given expression** | **Re-expressed form of given expression** |
| 1. *(x*–*3)(4x*–*3)* | = *(x-3)(4x-3)* |
| 2. *(x2+x*–*20)(3x2+2x*–*1)* | = *(x-4)(x+5)(3x-1)(x+1)* |
| 3. *(3x–1)(x2–x–2)(x+5)* | = *(3x-1)(x+1)(x-2)(x+5)* |
| 4. *(-x+3)2 +x(3x–9)* | = *(-x+3)(-x+3) + x(3x-9)*  = *(-1)(x-3)(-1)(x-3) + 3x(x-3)*  = *(x-3)(x-3) + 3x(x-3)*  = *(x-3)(x-3+3x)*  = *(x-3)(4x-3)* |
| 5. |  |

II(C) In Part I C, you made some conjectures based on numerical evaluations of expressions. Explain in what way the algebraic manipulations in Part II B supported (or not) each of those conjectures.

We re-expressed Expression4 in the form of Expression1—this supports the first conjecture in Part I C (as was established in the classroom discussion following Part I).

The same is true for the second conjecture of Part I C.

For any conjectures of Part IC not supported by your algebraic manipulations in Part IIB, how do you account for the discrepancy?

**Activity 1: Equivalence of Expressions**

**Lesson 2: Testing for equivalence of expressions using CAS**

**Part III (with CAS):**

**Testing for equivalence by re-expressing the form of an expression—using the EXPAND command**

The left-hand column of the table below contains the expressions from the previous lesson. Using your calculator, fill in the right-hand column with the expression produced by the EXPAND command (see F2 menu in the calculator).

Syntax: EXPAND(*expression*)

|  |  |
| --- | --- |
| Given expression | Result produced by EXPAND |
| 1. *(x*–*3)(4x*–*3)* | *4x2-15x+9* |
| 2. *(x2+x*–*20)(3x2+2x*–*1)* | *3x4+5x3-59x2-41x+20* |
| 3. *(3x–1)(x2–x–2)(x+5)* | *3x4+11x3-25x2-23x+10* |
| 4. *(-x+3)2 +x(3x–9)* | *4x2-15x+9* |
| 5. | *3x4+11x3-25x2-23x+10* |

## Classroom discussion of Part III

Definition of **equivalent expressions:**

We specify a set of admissible numbers for *x* (e.g., excluding the numbers where one of the expressions is not defined). If, for any admissible number that replaces *x*, each of the expressions gives the same value, we say that these expressions are equivalent on the set of admissible values.”

**Part IV (with CAS):**

**Testing for equivalence without re-expressing the form of an expression—using a test of equality**

It is possible to explore whether two expressions are equivalent without having to re-express their forms. An alternative approach is to use a CAS test of equality:

IV(A) Enter directly into your calculator’s entry line the equation formed by expressions 3 and 5:

*(3x–1)(x2–x–2)(x+5) =* 

1. What does the calculator display as a result?

|  |
| --- |
| “true” |

2. How do you interpret this result?

|  |
| --- |
| “true” indicates that the two expressions are equivalent on the set of real numbers, except x = -2 |

3. Use your calculator’s “with operator” (**|**) to replace *x* by –2 in the above equation. Interpret the result displayed by the calculator.

|  |
| --- |
| The CAS displays “false” because *x* = -2 is an inadmissible value (i.e., the expression on the right hand side is not defined at this value, so equality is nonsensical) |

## Classroom discussion of Part IV A

IV(B) Enter directly into your calculator’s entry line the equation formed from the two given expressions 2 and 3:

 = 

1. What does the calculator display as a result?

|  |
| --- |
| = |

1. How do you interpret this result?

|  |
| --- |
| This indicates that the two expressions are *not* equivalent |

## Classroom discussion of Part IV B

**Part V (with CAS): Testing for equivalence—using either CAS method**

Here is a new set of expressions:

|  |
| --- |
| Given expression |
| 1. |
| 2. |
| 3. |
| 4. |

V(A) Use your CAS to determine which of these expressions are equivalent. Use whichever CAS method you prefer. Show all your CAS work in the table provided below:

|  |  |
| --- | --- |
| What you enter into the CAS | Result displayed by CAS |
| EXPAND() | *3x2-10x+3* |
| EXPAND() | *x2+5x-24* |
| EXPAND() | *3x2-10x+3* |
| EXPAND() | *3x2-10x+3* |

V(B) Based on your work above, which are the equivalent expressions (don’t forget to specify the set of admissible values for *x*)? Please explain your decisions about equivalence.

Expressions 1, 3, and 4 are equivalent over all real numbers different from x = -2.

All three expressions are re-expressible in a common form, so they are equivalent.