Name:

Activity 7: Factoring and Solving Equations Involving

Expressions with Radicals

*Note to student*: The primary objective of this activity is that you come to view and employ factoring (taking out a common factor) as a tool for solving equations, particularly when used in conjunction with the “zero product theorem.”

*Here are some secondary objectives*:

* + To understand that factoring (taking out a common factor) can be applied not only to constants and variables, but also to algebraic expressions that can be taken as objects to operate upon;
  + To be able to reactivate, at a moment’s notice, the methods learned for solving linear and quadratic equations. You should be able to employ these methods when solving equations that are neither linear nor quadratic, per se;
* To understand that simplifying an equation by dividing both sides by some factor may lead to a loss of solutions. In equations in which such simplifications are possible, the strategy of isolating terms on one side of the equation and using the zero product theorem is generally a more effective solving method;
* To understand the necessity of verifying one’s solution to equations involving variables under the radical sign.

1. Suppose you were asked to solve this equation:

 (\*)

a) How would you proceed when faced with such a “monster”? (Don’t actually solve the equation, just state your general approach.)

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| A general approach is to treat as an object of algebraic manipulation; one that we can isolate in the given equation.  A recommended solution strategy is to group terms containing as a factor, and to then employ factoring and the zero-product theorem to solve the resulting equivalent equation.  Another solution strategy might be to divide both sides of the equation by , under the constraint that *x* > 4 (why?). This produces an equivalent equation that is linear in *x* and which can be easily solved. (What are the limitations of this strategy?) |

1.b) Using paper and pencil, see whether you can first solve the following equation that is somewhat analogous to the above monster:

*(y-2)3 –10(y-2) = y(y-2)* (\*\*)

*Hint*: Factoring (taking out a common factor) might be useful here.

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| Following the general approach outlined in 1a:  1. Subtract *y(y-2)* from both sides of (\*\*) to produce the equivalent equation  *(y-2)3 –10(y-2)-y(y-2) = 0*.  2. Factor *(y-2)* from each term to produce a second equivalent equation  *(y-2)[(y-2)2 –10-y] = 0*.  3. Simplify this third equation using algebra and factoring to produce the following sequence of equivalent equations:  *⇔ (y-2)[(y-2)2 –10-y] = 0*  *⇔ (y-2)[y2-4y+4 –10-y] = 0*  *⇔ (y-2)[y2-5y–6] = 0*  *⇔ (y-2)(y+1)(y-6) = 0*  4. Invoke the zero-product theorem to solve this last equivalent equation:  *(y-2)(y+1)(y-6) = 0 ⇔ y-2 = 0 or y+1 = 0 or y-6 = 0*    *⇔ y = 2 or y = -1 or y = 6* |

1.c) Compare your solution to equation (\*\*) with that obtained using the calculator’s SOLVE command. If the solutions obtained are different, verify your paper and pencil algebraic work. If the calculator produced an additional solution to the ones you found, determine which among the paper and pencil algebraic manipulations you used led to the loss of this additional solution. Please show all your work in the space below.

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| The CAS command “Solve(*(y-2)3 –10(y-2) = y(y-2), y)*” produces the display  “*y = 2 or y = -1 or y = 6*”.  Thus, the CAS obtained exactly the same solution set as that obtained above with paper and pencil algebra. |

#### Classroom discussion of Questions 1a, b, & c

2. a) On the basis of the strategies employed in solving the previous equation (\*\*), use paper and pencil to find the solution set of the following equation:

 (\*\*\*).

Show all your work in the space provided below.

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| Treat  as an object of algebraic manipulation:  1. Subtract *(3u-7)* from both sides of (\*\*\*) to produce the equivalent equation    *5()3 + 4 - (3u-7) = 0.*  2. Factor  out of each term to produce another equivalent equation  [*5()2* + *4 - (3u-7)*] *= 0.*  3. Simplify using algebra, to produce the equivalent equations  (*5u* + *4 - 3u + 7) = 0 ⇔*  *(2u +11) = 0.*  4. Invoke the zero-product theorem to solve this final equivalent equation:  *(2u +11) = 0⇔  = 0 or (2u +11) = 0*  *⇔ u = 0 or u = -11/2*  5. Since  does not define a real number for any value of *u* < 0, then *u = -11/2* is an inadmissible value. Thus, the solution set of (\*\*\*) is {0}. |

2. b) Substitute the values you obtained as solutions for equation (\*\*\*) using your calculator’s “with operator” (“**|**”). What does the calculator display as a result? Are there any solutions that you would discard? Why or why not?

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| Upon entering the syntax “ **|** *u* = {0, -11/2}” and pressing the ENTER key, the CAS displays the result “{true, false}”.  Thus, the CAS correctly recognizes that *u = -11/2* is not an admissible solution (since this value does not make the equation a true statement, over the real numbers). We must therefore discard *u = -11/2*, as explained in Part 2*a* above. |

##### Classroom discussion of Question 2

3. Continuing with paper and pencil, now try to solve the original equation (\*):

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Determine first the condition under which the solutions are admissible, given the radicals. Then, compare your solution with that produced by the calculator and discuss the validity of each value displayed. Show your work in the space provided below:

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| Since  defines a real number only for *x* ≥ 4, then only solutions that satisfy this condition are admissible.  We follow a solution strategy analogous to that used in Part 2a:  ⇔[5()2 + 11 - (2*x* + 1)] = 0  ⇔  [5(*x*-4)+ 11 - 2*x -* 1)] = 0  ⇔  [5(*x*-4)+ 11 - 2*x -* 1)] = 0  ⇔  (3*x-*10) = 0  ⇔  = 0 or (3*x-*10) = 0  ⇔ *x* = 4 or *x* = 10/3  Since *x* = 10/3 < 12/3 = 4, this solution is inadmissible. Therefore {4} is the solution set for the given equation.  The CAS “solve” command produces the same solutions, but verification with the CAS (using the “evaluate” operator) indicates that *x* = 10/3 is indeed inadmissible over the real numbers. |

##### Classroom discussion of Question 3

4. Challenge Problem

a) Solve the following equation using paper and pencil:

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Following the approach of the last several problems, we can first note that only solutions satisfying the condition 2*x*-1 ≥ 0, or equivalently *x* ≥ 1/2, are admissible.

We can solve the equation by grouping and factoring, treating  as an object to manipulate algebraically:



⇔  [()4 + 3()2 + 5 - (8*x*+7)] = 0

⇔  [(()2)2 + 3(2*x*-1) + 5 - 8*x* - 7)] = 0

⇔  [(2*x*-1)2 + 6*x* - 3+ 5 - 8*x* - 7)] = 0

⇔  [4*x*2 - 4*x* + 1 - 2*x* - 5] = 0

⇔  (4*x*2-6*x*-4) = 0

⇔ 2 (2*x*2-3*x*-2) = 0

⇔ 2 (2*x* + 1)(*x* - 2) = 0

⇔  = 0 or 2*x* + 1 = 0 or *x* - 2 = 0

⇔ *x* = 1/2 or *x* = -1/2 or *x* = 2

The value *x* = -1/2 is inadmissible since it does not satisfy the condition noted above.

b) What solutions does the calculator display for this equation? Discuss the validity of these solutions.

Using the “solve” command, the CAS displays the same three solutions obtained above. Only the value *x* = -1/2 is inadmissible, as noted above. This can be verified by using the evaluate operator with these three values and the original equation, wherein the CAS displays the result {true, false, true}. Unfortunately, the calculator displays true for the three values.