Activity 4: Rationalizing the Denominator of an Expression

Point of insertion: Volume 1, Chapter 2 of “Reflections 436” textbook, before section on Rationalization on page 114

# Content: Rationalization of the denominator of an expression.

# Objective: To enable students to rationalize a denominator, and to simplify expressions by using multiplication of conjugate forms.

**Note:** Rationalization is to be characterized as that algebraic operation that consists of eliminating the radical(s) appearing in the denominator of an expression. The conjugate of a sum of two terms is defined as the difference of these terms, and vice versa.

**Part I (20 minutes):**

**Calculator and paper& pencil activity**

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| --- |
| a-i) Enter the expression  into your calculator. What do you notice? |

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**Note to teacher.** Using the calculator, we get this result directly: .

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| a-ii) What paper and pencil calculation will produce the same result as the calculator’s (in part a-i above)? |

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**Note to teacher.** Whole-class discussion should highlight this definition: Rationalizing is the operation that consists of eliminating the radical(s) appearing in an expression’s denominator, without changing the expression’s value (i.e., multiplying the expression by a fraction that is equivalent to 1).

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**Calculator and paper & pencil activity**

I.b) The following activity continues the work on rationalizing denominators of expressions (answers are provided in the table below for the teacher’s benefit).

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| --- | --- | --- |
| Expression | Enter each expression into your calculator and write the result it displays | Paper & pencil work that transforms the original expression into the form produced by the calculator |
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**Note to teacher:** The whole-class discussion should recall the identity that students have already learned: . It will be important to discuss how to manipulate radicals and the possibility of obtaining –1 as a common factor (as in the last example above). The aim is to motivate multiplication of an expression by its conjugate.

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**Paper & pencil activity**

I. c) On the basis of strategies you used to rationalize denominators of the previous expressions, fill in the empty cells of the table below; using paper and pencil.

|  |  |
| --- | --- |
| Expression  ***(c ≥ 0 & d ≥ 0)*** | Manipulation that rationalizes the denominator of the given expressions |
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|  |  |

Explain why the restrictions *c ≥ 0 & d ≥ 0* are necessary when considering the given expressions above. What other restrictions, if any, apply?

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**Note to teacher.** In the two cases above, if the calculator were to be used, it would be found that it displays exactly the same result as the input expression – for reasons that are not obvious. In the whole-class discussion, therefore, promote paper-and-pencil work for these two expressions.

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**Part II (10 minutes):**

**Calculator and paper & pencil activity**

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| IIa) What paper & pencil manipulations will rationalize the denominator of the expression   ? |

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| IIb) What restrictions are necessary when considering the expression in (a)? Explain why. |

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| IIc) Enter the expression  into your calculator and write down the result it displays. |

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| IId) If the calculator displays a result that differs from the one you obtained in Part II a), what can you do to reconcile the two? |

**Note to teacher.** The whole-class discussion should be conducted so as to consolidate the work done thus far.

**Part III: A challenge**

**(paper & pencil activity)**

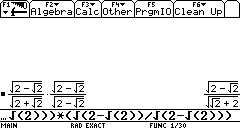
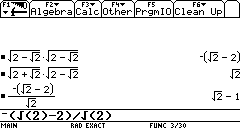
We wish to use a strategy similar to the one used thus far to rationalize the denominator of the expression .

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| IIIa) Enter the expression  into the calculator and write down the result it displays. What do you notice? |

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| IIIb) What paper & pencil calculation can you use to rationalize the denominator of the expression  ? |

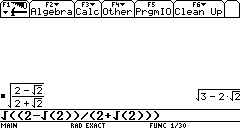
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**Note to teacher**. Here, the calculator does not simplify the given expression. We could propose that the multiplication be done in parts: First, the product of numerators , then the product of denominators , and, finally, divide the two results obtained. This produces the following displays:

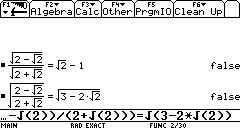
 

**Something to pay attention to: the CAS sometimes does not do everything**

If we do think that this activity is an appropriate one, there are a few things to be attentive to – one concerns the results produced by the calculator. There are numerous procedures that students can employ in this activity. For example, representing the division of the two radicals as one radical (with the division under the radical sign):



Now, it is possible that after a whole-class discussion students will insist on the equivalence of the expressions obtained (i.e.,  and ). If they use the calculator to verify this equivalence, they might be confronted with unexpected and incorrect results! The two tests displayed in the screen image below demonstrate what appears to be a calculator bug. We would thus not encourage students to verify these equivalences with the CAS.



On the other hand, the following equivalence is handled correctly by the CAS (see the screen image below). Nevertheless, it might be advisable to verify this equivalence with paper & pencil only.

