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## Model Theory and Applications

edited by L. Bélair, Z. Chatzidakis, P.D'Aquino, D. Marker, M. Otero, F. Point and A. Wilkie.



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Definable Sets in Valued Fields: 03C60;

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Tame Minimal Simple Groups of Finite Morley Rank and of Odd Type: 20E32, 03C60;

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Model Theory, Geometry and Arithmetic of the Universal Cover...: 03C45, 03C60, 14K15.

## Introduction

In the last few years model theorists have found deep connections between their subject and other areas of mathematics. Our goal in organizing the European Conference on Model Theory and Applications was to assess the state of the art, especially for the younger generation.

This conference was also the opportunity to acknowledge, on the occasion of his sixtieth birthday, the influence of Angus Macintyre in the development of our subject. This has been both immense and diverse and we feel it appropriate to mention a few highlights here.

Angus' research has links with algebra, geometry, number theory, asymptotics and even to computational complexity theory and randomized algorithms. His early successes included characterizations of uncountably categorical abelian groups and fields as well as results on the elementary theory of Banach algebras. He also completed the proof of a beautiful equivalence (the "if" direction being due to B H Neumann) between model theoretic and recursion theoretic notions in combinatorial group theory: a finitely generated group can be embedded in every algebraically closed group if and only if it can be recursively presented with solvable word problem. Soon after this period,

while still making deep contributions to the model theory of groups and rings and, indeed, of arithmetic (both to foundational questions involving Peano Arithmetic and its subtheories, and to the more application orientated “non-standard number theory”), he established his quantifier elimination result for  $p$ -adic fields. As we all now know, this was a pivotal moment in applied model theory and, through the work of Denef and many others, Angus’ insights at that time continue to have deep applications in  $p$ -adic diophantine geometry via the theory of Poincaré series. It does seem remarkable that the original work was done almost thirty years ago. In fact, we list below a chronological selection of his papers so that the reader will be able to appreciate more fully the ramifications of Angus’ ideas on what we now call the model theory of fields and his dominant role within it.

Almost all of these papers are extremely familiar to the editors of this volume, but to those not brought up under Angus’ influence may we further single out his definitive work on PAC and regularly closed fields, his synthesis of Ax’s methods for finite fields with difficult analytic number-theoretic results to obtain information on primes in nonstandard models of systems of arithmetic and, of course, his work on exponentiation. Many of his well known results on the decidability and axiomatizability of the real exponential function, and on exponential series, occur in joint papers in the nineties, but it should be pointed out that he had already originated the study of the  $p$ -adic analogue many years before this. Also, he was the first to truly believe, in the early eighties, that the real case (Tarski’s problem) was ready for a serious attack: subsequent progress would certainly not have been made were it not for his early papers, the input of numerous preprints and his constant encouragement.

You will also see that his more recent research is just as wide-ranging. It covers generic automorphisms of algebraically closed fields, the model theory of analytic functions and its application to sigmoidal neural networks, formal models for exponential asymptotics and model-theoretic aspects of the Weil conjectures for finite fields.

But his influence in model theory goes beyond any list of his achievements. It is also due to the excitement he conveys, whether it be in his lectures or just in everyday conversations at meetings around the world. He is well known

in the community for his accessibility to discuss mathematics, and the circle of his friends and students has always benefited from his exceptional energy, his enthusiasm and generosity. There is no doubt that through this personal contact he has helped turn our subject into the thriving area that it now is.

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